Heat Transfer to Newtonian and Non-Newtonian Fluids in mechanically Agitated Vessel

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**Abstract:** Heat transfer to Newtonian (water) and power law Non-Newtonian (1%, 2% and 4% aqueous CMC solution) fluids in the agitated vessel is investigated. The data have been obtained for fluid agitated by three marine agitators of 7.5, 12.7 and 18.35 cm diameter respectively. The heat transfer coefficient has been calculated using Wilson graphical method with modification suggested by Om Prakash et al. The heat transfer data for agitated water and 1, 2 and 4% aqueous CMC solutions for impeller diameters have been correlated by the equation $N_{Nu} = 0.302 N_{Re}^{0.13} Pr_{a}^{-0.1} D_{r}^{0.1}$ with standard deviation of 8.03%.

**Keywords:** Agitator, Impeller, Non-Newtonian, Power Law Fluid, Power Law.

I. INTRODUCTION

From the available literature it is seen that the average heat transfer coefficient depends on the geometrical and physical factors. The correlation between them is not so simple because of complex relationship between large numbers of variables involved. The heat transfer rate in an agitated vessel, either from the vessel wall or from the wall of coils, depends upon velocity and temperature profiles near the wall. The velocity profile itself, which affects the temperature profile, depends on geometrical factors, such as the impeller diameters and its nature, position and speed, the diameter of the fluid reservoir, the diameter and pitch of the coils, the diameter of the coil tube, the fluid height in the tank and also on the position and dimensions of the baffles. The physical parameters are the physical properties of the fluids, specially, the velocity, the thermal conductivity, density, specific heat and their variation with the temperature. Physical parameters are normally grouped in terms of prandtl number, Reynolds number and viscosity correction factors whereas geometrical properties are considered in Reynolds number and various geometric dimensionless groups.

The non-linear relationship between shear stress and shear rate for non-Newtonian materials makes the development of the heat transfer correlation more complex. For flow of such materials difficulties arise in specifying the Reynolds number, Prandtl number and viscosity ratio. Therefore an appropriate choice of viscosity is necessary to obtain suitable correlation for heat transfer. Carreau, Charest and Cornellio (1) studied heat transfer to pseudoplastic fluids in jacketed vessel using turbine agitator and employed a generalized Reynolds number analogous to that of laminar flow in pipes while Reynolds number was defined by using differential viscosity at high shear rate. The flow behavior of the fluids in agitated vessel is not correctly and adequately described by the Reynolds number used by them. Thus some variation exists in the final correlation in the coefficients for Newtonian and non-Newtonian fluids. Gluz and Pevlushenko (2) used a viscosity based on average shear rate corresponding to shear rate at the surface of a rotating cylinder, and the Reynolds number defined by Gluz et al. describes the flow behavior in a better manner compared to that used by Carreau et al.

Hagedorn and Salamone (3) considered the flow pattern at the vessel wall for power law fluids and presented the correlation on the basis of dimensional analysis of basic equations. OC Sandall et al(7) also considered the shear rate at the wall and correlated the data on Newtonian and non-Newtonian fluids for anchor agitator assuming shear rate proportional to ratio of agitator tip speed and the clearance between the agitator tip and the wall. Many other workers (5,6,7,8,9,10,11,12) have made efforts to correlate the similar data by using average shear rate derived from agitator power measurements following the methods of Metzner and Otto(13) and Calderbank and Moo Young(14,15). For power law fluids, shear stress and shear rate relationship at the rotor of the bob viscometer in an infinite fluid (considered as the surface of the cylinder rotating is on infinite fluid) is given by

$$\tau_r = \left[ r \frac{d(\theta r)}{dr} \right] \tau_n = K(4\pi N/n)^{n}$$

and the relationship for generalized power law may be stated as
For power law fluids \((n^+ = n)\) various viscosity and Reynolds number expressions may be defined as follows:

\[
\tau_d = K (4\pi N)^{n^+} \tag{2}
\]

\[
\left[ \frac{d\left(\frac{\partial }{\partial r}\right)}{dr} \right]_s = \frac{4\pi N}{n^+} \tag{3}
\]

And

\[
K'' = K/(n)^n \tag{4}
\]

For power law fluids \((n^+ = n)\) various viscosity and Reynolds number expressions may be defined as follows:

Pseudo shear viscosity

\[
\mu'' = \frac{\tau_d}{4\pi N} = \frac{K}{n^+} (4\pi N)^{n^+ - 1} \tag{5}
\]

and the corresponding Reynolds number

\[
N_{Re}'' = \frac{D_a U_a \rho}{\mu''} = \frac{(n)^n D_a^2 N^{2-n} \rho}{K(4\pi N)^{n^+ - 1} n^{n^+ - 2}} \tag{6}
\]

Apparent or shear viscosity may be written as

\[
\mu'' = \frac{\tau_s}{\int \frac{d\left(\frac{\partial }{\partial r}\right)}{dr} dr} = n(1-n) K (4\pi N)^{n^+ - 1} \tag{7}
\]

\[
N_{Rea}'' = \frac{D_a^2 N^{2-n} \rho}{n^{1-n} K(4\pi N)^{n^+ - 1} (n)^{n^+ - 2}} \tag{8}
\]

Differential viscosity

\[
\mu'' = \frac{\tau_s}{\int \frac{d\left(\frac{\partial }{\partial r}\right)}{dr} dr} \tag{9}
\]

or

\[
\mu'' = \frac{d\ln \tau_s}{\int \frac{d\left(\frac{\partial }{\partial r}\right)}{dr} dr} \tag{10}
\]

From equations (5), (7) and (9) one obtains

\[
\mu'' = n \mu'' = n^2 \mu'' \tag{11}
\]

and relationship between various Reynolds numbers \(N_{Re}''\), \(N_{Rea}''\) and \(N_{Red}''\) on the basis of pseudo shear, shear and differential viscosity, respectively, may be written as

\[
N_{Rea}'' = n N_{Re}'' = n^2 N_{Red}'' \tag{12}
\]

Similarly corresponding Prandtl numbers may be defined as

\[
N_{Pr}'' = n N_{Pr}'' = n^2 N_{Pr}'' \tag{13}
\]

Where

The analysis of the flow pattern around the mixing impeller is not so simple. A compromise between experimental and physical pictures of the flow pattern is needed to obtain suitable parameters for the correlation. Let us consider the case of flat plate submerged in a fluid flowing in a laminar fashion. If the thickness of the flat plate is small compared to width, most of the drag on the plate will be due to pressure difference between front and rear of the plate which is caused by the motion of the fluid near the surface. Shear stress causing any drag will be negligible. Now let us consider the flow situations around the flat blade of turbine or paddle rotating in a fluid. At low speed of rotation the flow pattern may be considered similar to that of flat plate submerged in a fluid, in motion. At higher speed, with more number of blades the extent of the separation of flow will decrease and the viscous energy dissipation becomes the controlling factor. Thus the use of dimensionless groups defined by visualizing the system as a cylinder of diameter equal to that of blade rotating in an infinite fluid medium will be better than compared to the paradoxical definitions of Reynolds number and viscosity analogous to pipe flow.

For heat transfer from the wall of the vessel agitated by a turbine agitator, the fluid flow along the wall of the vessel is upward and the resistance to the heat transfer is mainly controlled by the viscous sub-layer along the wall, with a large velocity and temperature gradient in radial direction. The heat transferred through the viscous sub layer is completely mixed with the bulk of fluid between the sub layer and the impeller. Under steady state condition the momentum, mass and energy equations for power law fluids in the region near the wall may be simplified and presented in the following form:

\[
p \rho u_r = \frac{\partial u_z}{\partial z} - \frac{1}{r} \frac{\partial}{\partial r} \left( r \rho u_r \right) = 0 \tag{14}
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \rho u_r \right) = 0 \tag{15}
\]

\[
\frac{\partial}{\partial r} \left( \frac{\partial T_r}{\partial r} \right) = \frac{K_r}{\rho C_p} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_r}{\partial r} \right) \right] \tag{16}
\]

The viscous sub layers thickness \(\delta\) will be very small compared to the vessel diameter. As the Reynolds number, \(N_{Rea}''\), increases the thickness of the viscous sub layer decreases. The \(\delta\) will be a function of \(D_a\) and \(N_{Rea}''\) and \(U_o\) will be function of \(N_{Rea}''\) and characteristic velocity, \(\pi D_a N\).

The following dimensionless variables, now can be defined as

\[
r^* = \frac{r}{D_a} \tag{17}
\]

\[
u_r^* = \frac{u_r}{D_a N} \tag{18}
\]

\[
Z^* = \frac{Z}{D_a} \tag{19}
\]

\[
p^* = \frac{(p - p^o)/\rho (\pi D_a N)^2}{p^o} \tag{20}
\]

\[
T^* = \frac{(T - T_o)}{T_s - T} \tag{21}
\]

Substituting the above dimensionless variables in equations (14) and (16) it is observed that the velocity gradient \(\frac{\partial u_r}{\partial r}\) is a function of \((N_{Rea}''^{(m+1)})^{-1}\) and the temperature gradient \(\frac{\partial T^*}{\partial r}\) is a function of \((N_{Rea}''^{(m+1)})^{-1}\).

Using Fourier law of heat conduction and Newton’s law of cooling, the expression for total heat flux and heat transfer
Heat Transfer to Newtonian and Non-Newtonian Fluids in mechanically Agitated Vessel

The final expression for Nusselt number may be arranged.

\[ N_{uj} = h_j D_f / K = \frac{N_{Rea}^{m}}{H} \int_{0}^{H} \frac{\partial T}{\partial Y} \, dY \]  

(22)

Thus it is seen that \( N_{uj} \) is a function of \( N_{Rea}^{m} \), \( N_{Pra}^{m} \) and length ratio \( H/D_f \). If non-isothermal correlation factor is included the correlation takes the form

\[ N_u = C_1 N_{Rea}^{b1} N_{Pra}^{b2} (\mu_{ab} / \mu_{av})^{0.14} \]  

(23)

Retaining conventional values of Reynolds and Prandtl number indices \( b_1=2/3 \) and \( b_2=1/3 \) respectively and sieder tate correction factor \( (\mu_{ab} / \mu_{av})^{0.14} \), the equation 23 takes the form

\[ N_{uj} = C_1 (N_{Rea}^{m})^{2/3} (N_{Pra}^{m})^{1/3} (\mu_{ab} / \mu_{av})^{0.14} \]  

(24)

Most of the data available for heat transfer in agitated vessel are for Newtonian fluids only. The similar data for non-Newtonian fluid is almost meager. Therefore experimental study was carried out to obtain data for heat transfer to purely viscous non-Newtonian fluids in agitated vessel.

II. EXPERIMENTAL

The experimental setup considered of a flat bottom test vessel of 45.25 cm inner diameter and 60 cm. heights made from 1/8 inch thick copper sheet. The vessel was jacketed from 1/8 inch thick GI sheet. A rectangular tank of about 300 litre capacity filled with heaters was used to heat the water to a pre determined temperature which in turn was circulated through the jacket (annular space around the test vessel) with the help of a centrifugal pump. The fluid in the test vessel was agitated by marine type agitator fitted in the centre of the coil. The agitator shaft was driven at a known speed by a 2HP electric motor through a reduction gear assembly. Provision was made for replacement of impeller of desired shape and size. The water in the hot water tank was heated and the temperature of fluid was brought to the desired level. The temperature was controlled to a pre determined value with the help of temperature controller. The water circulation were then started in the jacket and adjusted by means of regulating valves and bypasses. The agitator was then started at a fixed rpm. At steady state condition the inlet and outlet water temperatures in the jacket, mass ratio of flow water from jacket test fluid from, rpm of the agitator and temperatures of fluid in the agitated vessel and that of water in the storage tank were noted.

The test fluid side wall temperatures of the test vessel were noted at different locations and at different heights from its bottom with the help of copper constantan thermocouples. The readings were duplicated to ensure the steady state and to eliminate any error in measurement. Similar measurements were made by varying the flow rate in the jacket and then by varying the rotational speed of the agitator. Above procedure was repeated for all the fluids. The heat transfer characteristics of five fluids, viz., water and four aqueous CMC solutions of concentration 0.5, 1, 0.2 and 4% by weight have been investigated. The rheological properties were determined with the help of capillary tube viscometer. All the CMC solutions were found to be pseudoplastic in nature obeying power law relation. The flow behavior indices were found to be 0.937, 0.851, 0.793, and 0.698 for 0.5%, 1%, 2% and 4% aqueous CMC solutions respectively. Thermal conductivity of CMC solutions were determined by comparative concentric cylinders and were found to be equal to that of water. Specific heats of the solutions, as measured by calorimetric method, were found to be nearly equal to that of water.

III. RESULT AND DISCUSSION

In order to evaluate constant \( C_1 \) and Reynolds and Prandtl number indices \( b_1 \) and \( b_2 \) respectively the viscosity \( \mu_a \) and following dimensionless groups were calculated.

\[ N_{uj} = h_j D_f / K \]  

(25)

Liquid film heat transfer coefficient \( h_j \) were calculated from over all coefficients \( U_j \) using Wilson graphical method modified by Om Prakash et al.\n
\[ \mu'' = n^{-1} \, (K) \, (4 \pi N)^{n-1} \]  

(26)

Reynolds and Prandtl number may be written as

\[ N_{Rea}^{m} = \frac{D_a^{2} \pi D_j N}{K n^{1-n} \, 4 \, \pi \, n^{1-n}} \]  

(27)

A. Effect of Prandtl Number

Prandtl number influence is shown in fig.1 where \( N_{Nuj}^{m} / N_{Rea}^{m} \) is plotted against \( N_{Nuj}^{m} \) and the influence of Reynolds number is eliminated by dividing \( N_{Nuj}^{m} \) values by \( N_{Rea}^{m} \).

![Fig.1. Jacket to agitated fluid heat transfer correlation (effect of prandtl number).](image)

The exponent of \( N_{Rea}^{m} \) was choosen as 2/3 from the previous experience. Prandtl number of polymeric solutions obeying power law varies with the speed of the agitator and is found to vary from a minimum of 4.9 for water to a maximum of 850 for 4% CMC solution in the present investigation. The average line through the data points of fig.2 gave the exponent \( b_2 \) of the Prandtl number equal to 1/3 which agrees with Prandtl number exponent in heat transfer correlation for Newtonian fluids.
Fig. 2. Heat transfer correlation for agitated non-Newtonian fluids (effect of Reynolds number).

B. Effect of Impeller Diameter

For studying the effect of impeller diameter, the heat transfer measurements were made with three impeller of diameter 7.5, 12.7 and 18.35 cm respectively. Although the diameter of impeller, \( D_a \) shows its effect is Reynolds number, an attempt has been made to find the effect of \( D_a/D_T \) ratio on Nusselt number. Fig. 3 shows a correlation between \( (N_{uj}/N_{Pra}^{1/3} N_{Rea}^{2/3}) \) and \( D_a/D_T \) in which data of three fluids for \( D_a/D_T \) ratio 0.166, 0.282 and 0.403 have been plotted. Almost negligible effect of \( D_a/D_T \) is observed possibly because of very small range of \( D_a/D_T \) ratio covered in the present work.

Fig. 3. Heat transfer correlation for jacket to agitated fluids: effect of agitator diameter.

However, a mean line through data point shows the index of \( D_a/D_T \) equal to 0.1. Thus the overall effect of \( D_a \), including its presence in Reynolds number, gives

\[ h_j \propto D_a^{1.433} \]  

(25)

Finally \( N_{uj}/N_{Pra}^{1/3} (D_a/D_T)^{0.1} \) is plotted against Reynolds number \( N_{Rea}^{2/3} \) in fig.4 which includes data for water and 1, 2 and 4% CMC solution. The method of least square gives the equation

\[ N_{uj} = 0.302 N_{Rea}^{2/3} N_{Pra}^{1/3} (D_a/D_T)^{0.1} \]  

(26)

Fig. 4. Heat transfer for jacket to agitated Newtonian and non-Newtonian fluids (effect of Reynolds number).

Fig. 5 compares the experimental Nusselt numbers with those calculated from equation 26. The standard deviation of 50 data points from equation 26 is found to be 8.03%.

Fig. 5. Comparison between experimental and calculated values of nusselt number \( N_{uj} \) (heat transfer from jacket to agitated fluids).

IV. CONCLUSION

- The Wilson plot of \( 1/h_j \) versus \( R_{rea}^{2/3} \) is appropriate for the determination of individual heat transfer coefficient.
- For small temperature driving forces the non thermal correction is negligible.
- The heat transfer data for agitated Newtonian and non-Newtonian fluids have been successfully correlated by the viscosity of the fluid evaluated at the impeller tip having a cylinder of diameter equal to that of impeller rotating in an infinite fluid data of 1, 2 and 4% CMC for three impeller diameters have been correlated by the equation

\[ N_{uj} = 0.302 N_{Rea}^{2/3} N_{Pra}^{1/3} (D_a/D_T)^{0.1} \]  

(26)

It is interesting to note that all data of both Newtonian and non-Newtonian fluids are in excellent agreement with equation 26 for \( 290 < N_{Rea} < 1.4 \times 10^7 \); \( 4.9 < N_{Pra} < 850 \) and \( 0.166 < (D_a/D_T) < 0.403 \).
Heat Transfer to Newtonian and Non-Newtonian Fluids in mechanically Agitated Vessel

With standard deviation of 8.03% for 290< N_Re<1.4x10^4; 4.9 < N_P<850 and 0.166< (D/D_T) < 0.403. Using the above concepts of Reynolds and Prandtl numbers it is also possible to correlate the available published data for other non-Newtonian fluids obtained with different impeller geometries.

V. NOMENCLATURE

1. \( \text{c N}_{\text{Nuj}} \) Nusselt number, \( h D_j/k \)
2. \( N'' \text{Rea} \) Reynolds number defined by equation 8
3. \( N' \text{Pr} \) Prandtl number defined by equation
4. \( D_a \) agitator diameter, cm
5. \( D_T \) diameter of the agitated vessel
6. \( \tau_s \) shear stress, gm(f)/cm^2
7. \( r \) radial distance, cm
8. \( u_o \) outlet velocity
9. \( K \) consistency index, gm sec^{-2}/cm
10. \( N \) speed (r.p.m)
11. \( N'' \) flow behavior index
12. \( K'' \) consistency index, gm sec^{-2}/cm
13. \( n'' \) generalized flow behavior index
14. \( \mu'' \) effective viscosity at the impeller tip defined by equatio [11]
15. \( N_{\text{Re}}'' \) Reynolds number defined by equation [6]
16. \( \dot{U}_s \) superficial velocity
17. \( \dot{P} \) pressure, gm(f)/cm^2
18. \( \mu_s \) effective viscosity
19. \( \mu_a'' \) apparent viscosity at the impeller tip defined by equation[13]
20. \( \theta \) function of time, sec
21. \( \mu_d \) differential viscosity, gm/cm sec
22. \( c_p \) specific heat, cal/gm°C
23. \( k \) thermal conductivity
24. \( N_{\text{Pr}}'' \) prandtl number defined by equation [7]
25. \( k \) thermal conductivity, cal/cm sec°C
26. \( \delta \) boundary layer thickness, cm
27. \( T_i \) inlet temperature
28. \( T_s \) surface temperature
29. \( P^0 \) reference pressure, gm(f)/cm^2
30. \( \dot{h} \) heat transfer coefficient for jacketed vessel wall to fluid, Kcal/hr m^2°C
31. \( H \) height of the fluid level in the vessel,cm
32. \( b_1, b_2, b_3 \) constants in equation [23]
33. \( \mu_{ab} \) shear or apparent viscosity at bulk temperature
34. \( \mu_{sw} \) shear or apparent viscosity at the wall temperature
35. \( c_1 \) constants in equations [23]
36. \( U_j \) jacket overall heat transfer coefficient, Kcal/hr m^2

VI. REFERENCES