International Journal of
Scientific Engineering and Technology Research

ISSN 2319-8885
Vol.02,Issue.19,
December-2013,
Pages:2208-2217

www.semargroups.org,
www.ijsetr.com

Differential Equations and their Applications

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Abstract: In this project, many of the laws of nature – in physics, chemistry, biology, engineering and astronomy find their most natural expression in the language of differential equations. In other words, differential equations are the language of nature. Applications of differential equations also abound in mathematics itself, especially in geometry and harmonic analysis and modeling. Differential equations occur in economics and systems science and other fields of mathematical science. Many physical and engineering problems when formulated in the mathematical language give rise to partial differential equations. Besides these, partial differential equations also play an important role in the theory of elasticity, hydraulics etc. Since the general solution of a partial differential equation in a region R contains arbitrary constants or arbitrary functions, the unique solution of a partial differential equation corresponding to a physical problem will satisfy certain other conditions at the boundary of the region R. These are known as boundary conditions. When these conditions are specified for the time t = 0, they are known as initial conditions. A partial differential equation together with boundary conditions constitutes a boundary value problem. In the applications of ordinary linear differential equations, we first find the general solution and then determine the arbitrary constants from the initial values. But the same method is not applicable to problems involving partial differential equations. Most of the boundary value problems involving linear partial differential equations can be solved by the method of separation of variables. In this method, right from the beginning, we try to find the particular solutions of the partial differential equation which satisfy all or some of the boundary conditions and then adjust them till the remaining conditions are also satisfied. A combination of these particular solutions gives the solution of the problem. In this thesis, we are solving variety of differential equation in the field of applied sciences and engineering and also finding how the solution of differential equation provide the agreement with real life problems. In this thesis, we are describing not only ordinary differential equation but also partial differential equation.

Keywords: Partial Differential Equation, Ordinary Differential Equation(ODE).

I. INTRODUCTION

A. Introduction to Differential Equation

A differential equation is an equation relating some function *f* to one or more of its derivatives. An example is

$$\frac{d^2}{dx^2}f(x) + 2x\frac{d}{dx}f(x) + f(x) = \sin x \qquad (1)$$

It is obvious that this particular equation involves a function f together with its first and second derivatives. Any given differential equation may or may not involve f or any particular derivative of f. But, for an equation to be a differential equation, at least some derivative of f must appear. The objective in solving an equation like Equation (1) is to find the function f. Thus we already perceive a fundamental new paradigm: When we solve an algebraic equation, we seek a number or perhaps a collection of number, but when we solve a differential equation we seek one or more functions. Many of the laws of nature – in physics, in chemistry, in biology, in engineering, and in astronomy – find their most natural expression in the language of differential equations. Putting in other words, differential equations are the language of nature. Applications of differential equations also abound in mathematics itself, especially in geometry and harmonic analysis and modeling. Differential equations occur in economics and systems science and other fields of mathematical science. It is not difficult to perceive why differential equation arises so readily in the sciences. If $y = f(x)$ is a given function, then the derivative df /dx can be interpreted as the rate of change of f with respect to x. In any process of nature, the variables involved are related to their rates of change by the basic scientific principles, that govern the process that is, by the laws of nature. When this relationship is expressed in mathematical notation, the result is usually a differential equation.

1. Ordinary Differential Equation

An ordinary differential equation (ODE) is a differential equation in which the unknown function (also known as the dependent variable) is a function of a single independent variable. In the simplest form, the unknown function is a real or complex valued function, but more generally, it may be vector-valued or matrix-valued: this corresponds to considering a system of ordinary differential equations for a single function. Ordinary differential equations are further classified according to the order of the highest derivative of the dependent variable with respect to the independent variable appearing in the equation. The most important cases for applications are first-order and second-order differential equations. For example, Bessel's differential.

$$x^2\frac{d^2y}{dx^2} + \frac{dy}{dx} + (x^2 - \alpha^2)y = 0 \qquad (2)$$

2. Partial Differential Equation

In Mathematics, a partial differential equation (PDE) is a differential equation that contains unknown multivariable functions and their partial derivatives. Partial differential equations (PDEs) are used to formulate problem involving functions of several variables, and are either solved by hand, or used to create a relevant computer model. PDEs can be used to describe a wide variety of phenomena such as sound, heat, electrostatics, electrodynamics, fluid flow, or elasticity. These seemingly distinct physical phenomena can be formalized identically in terms of PDEs, which shows that they are governed by the same underlying dynamic. Just as ordinary differential equations often model one-dimensional dynamical systems, partial differential equations often model multidimensional systems. PDEs find their generalization in stochastic partial differential equations.

3. Linear Differential Equations:

A linear differential equation is any differential equation that can be written in the following form.

$$a_n(t)y^{(n)}(t) + a_{n-1}(t)y^{(n-1)}(t) + \cdots + a_0(t)y(t) = g(t) \qquad (3)$$

The important thing to note about linear differential equations is that there are no products of the function, y(t), and its derivatives and neither the function or its derivatives occur to any power other than the first power.
The coefficients $a_0(t), \dots\dots\dots, a_n(t)$ and g(t) can be zero or non-zero functions, constant or non-constant functions, linear or non-linear functions. Only the function, y(t), and its derivatives are used in determining if a differential equation is linear.

4. Non-Linear Differential Equations:

If a differential equation cannot be written in the form, (3) then it are called a non-linear differential equation.

5. Homogeneous Ordinary Differential Equation:

A linear ordinary differential equation of order *n* is said to be homogeneous if it is of the form and there is no term that contains a function of *x* alone.

$$a_n(x)y^n + a_{n-1}(x)y^{n-1} + \dots a_1(x)y' + a_0(x)y = 0 \qquad (4)$$

Where $y = \frac{dy}{dx}$ i.e., if all the terms are proportional to a derivative of y (or y itself) and there is no term that contains a function of *x* alone. However, there is also another entirely different meaning for a first-order ordinary differential equation. Such an equation is said to be homogeneous if it can be written in the form.

$$\frac{dy}{dx} = F\left(\frac{y}{x}\right) \qquad (5)$$

Such equations can be solved in closed form by the change of variables $u = y/x$ which transforms the equation into the separable equation

$$\frac{dx}{x} = \frac{du}{F(u) - u} \qquad (6)$$

6. Non-Homogeneous Differential Equations

Non-homogeneous differential equations are the same as homogeneous differential equations, except they can have terms involving only *x* (and constants) on the right side, as in this equation:

$$\frac{d^4y}{dx^4} + x\frac{d^{2y}}{dx^2} + y^2 = 6x + 3 \qquad (7)$$

You also can write non-homogeneous differential equations in this format

$$y'' + p(x)y' + q(x)y = g(x) \qquad (8)$$

The general solution of this non-homogeneous differential equation is

$$y = c_1y_1(x) + c_2y_2(x) + y_p(x) \qquad (9)$$

In this solution, $c_1y_1(x) + c_2y_2(x)$ is the general solution of the corresponding homogeneous differential equation:

$$y'' + p(x)y' + q(x)y = 0 \qquad (10)$$

And $y_p(x)$ is specific solution to the non-homogeneous equation.

7. Original Differential Equation:

Consider the second order differential equation

$$\frac{d^2y}{dx^2} = f\left(x, y, \frac{dy}{dx}\right) \qquad (11)$$

By putting $\frac{dy}{dx} = z$, it can reduced to two first order simultaneous differential equations

$$\frac{dy}{dx} = z \text{ and } \frac{dy}{dx} = f(x, y, x) \qquad (12)$$

Which can be solved easily because equation (11) is second order differential equation, in this way it is converted in to first order simultaneous differential equation so we can solve equation (12) easily. And system of equation (11) is known as original differential equation.

II. REVIEW OF LITERATURE

Berezman *et al.* (1986) [3] had published an article "Calculation of the Eigen values of Mathieu's equation with a complex parameter". In this article he gave an effective numerical algorithm suggested for calculating the Eigen values of Mathieu's differential equation when the parameter of the equation takes complex values from a fairly wide range of variation. The algorithm is based on using the theory of continued fractions. The efficiency of the algorithm is verified by a series of numerical experiments and by comparing them with known numerical data. McCoy and Boersma (1986) [22] stated that the axial growth of plant tissue obeys the physical laws of energetic during deformation of a continuous medium. The concept of biological energy conservation was employed to formulate a mathematical model of axial plant growth. The model was derived from a statement of the exchange of the thermodynamic potential energy with the kinetic energy of deformation. This derivation does not invoke a force balance analogy with simple mechanical systems and has no turgor dependence. The derivatives with respect to tissue strain of the turgor, osmotic potential and extent of the biosynthetic reactions, therefore, all participate in the performance of the work of growth. The model formulation is unique to plant growth studies since it combines principles of mechanical energy conservation during deformation with a chemical thermodynamic description of the potential energy. The concept that the change of the thermodynamic potential energy performs the work of deformation is more general and applicable to biological systems than the currently employed force balance approach.

J.C. Butcher (1992) [6] proposed the role of orthogonal polynomials in numerical ordinary differential equations. Orthogonal polynomials have many applications to numerical ordinary differential equations. Some of these, especially those connected with the quadrature formulae on which many differential equation methods are based, are to be expected. On the other hand, there are many surprising applications, quite unlike traditional uses of orthogonal polynomials. This paper surveys many of these applications, especially those related to accuracy and implement ability of Runge-Kutta methods. Stevens *et al.* (2009) [28] stated that an extension was proposed to the Local Hermitian Interpolation (LHI) method; a meshless numerical method based on interpolation with small and heavily overlapping radial basis function (RBF) systems. This extension to the LHI method used interpolation functions which themselves satisfy the partial differential equation (PDE). In this way, a much improved reconstruction of partial derivatives was obtained, resulting in significantly improved accuracy in many cases. The implementation algorithm was described, and was validated via three convection diffusion- reaction problems, for steady and transient situations. A Crank-Nicolson implicit time stepping technique was used for the time-dependent problems. A form of 'analytical upwinding' was implicitly implemented by the use of the partial differential operator of the governing equation in the interpolation function, which included the desired information about the convective velocity field.

Abbas et al. (2010) [26] described "Darboux problem for impulsive partial hyperbolic differential equations of fractional order with variable times and infinite delay". He dealt with the existence of solutions to impulsive partial functional differential equations with impulses at variable times and infinite delay, involving the Caputo fractional derivative. This works was considered by using the non-linear alternative of Leray-Schauder type. Wen *et al.* (2010) [32] published a paper "Dissipativity and asymptotic stability of non-linear neutral delay integro-differential equations". It was concerned with the dissipativity and asymptotic stability of the theoretical solutions of a class of non-linear neutral delay integro-differential equations (NDIDEs). They first gave a generalization of the Halanay inequality which played an important role in the study of dissipativity and stability of differential equations. Then, they applied the generalization of the Halanay inequality to NDIDEs and the dissipativity and the asymptotic stability results of the theoretical solution of NDIDEs. From a numerical point of view, it was important to study the potential of numerical methods in preserving the qualitative behavior of the analytical solutions. It provided the theoretical foundation for analyzing the dissipativity and the asymptotic stability of the numerical methods when they were applied to these systems was validated.

Han et al. (2013) [17] published an article "A partial differential equation formulation of Vickrey's bottle-neck model: methodology and theoretical analysis". The continuous-time Vickrey model can be described by an ordinary differential equation (ODE) with a right- hand side which is discontinuous in the unknown variable. Such a formulation induced difficulties with both theoretical analysis and numerical computation. It was widely suspected that an explicit solution to this ODE does not exist. They advanced the knowledge and understanding of the continuous-time Vickrey model by reformulating it as a partial differential equation (PDE) and by applying a variational method to obtain an explicit solution representation. Such an explicit solution was then shown to be the strong solution to the ODE in full mathematical rigor. This methodology also leaded to the notion of generalized Vickrey model (GVM), which allowed the flow to be a distribution, instead of an integrable function. This feature of traffic modeling is desirable in the context of analytical dynamic traffic assignment (DTA). The proposed PDE formulation provided new insights into the physics of the Vickrey model, which leaded to a number of modeling extensions as well as connection with first-order traffic models such as the Lighthill-Whitham-Richards (LWR) model. The explicit solution representation also leaded to a new computational method.

Tohidi et al. (2013) [29] published an article "A collocation method based on Bernoulli operational matrix for numerical solution of generalized pantograph equation". This paper presents a direct solution technique for solving the generalized pantograph equation with variable coefficients subject to initial conditions, using a collocation method based on Bernoulli operational matrix of derivatives. Only small dimension of Bernoulli operational matrix is needed to obtain a satisfactory result. Numerical results with comparisons are given to confirm the reliability of the proposed method for generalized pantograph equations.

III. METHODOLOGY

The differential equation is an equation in which dependent variable independent variable and derivative of differential equation occur simultaneously. Many physical and engineering problems when formulated in the mathematical language give rise to partial differential equations. Besides these, partial differential equations also play an important role in the theory of Elasticity, Hydraulics etc. Since the general solution of a partial differential equation in a region R contains arbitrary constants or arbitrary functions, the unique solution of a partial differential equation corresponding to a physical problem will satisfy certain other conditions at the boundary of the region R. These are known as boundary conditions When these conditions are specified for the time t = 0, they are known as initial conditions. A partial differential equation together with boundary conditions constitutes a boundary value problem. In the applications of ordinary linear differential equations, we first find the general solution and then determine the arbitrary constants from the initial values. But the same method is not applicable to problems involving partial differential equations. Most of the boundary value problems involving linear partial differential equations can be solved by the method of separation of variables. In this method, right from the beginning, we try to find the particular solutions of the partial differential equation which satisfy all or some of the boundary conditions and then adjust them till the remaining conditions are also satisfied. A combination of these particular solutions gives the solution of the problem.

A. Separation of Variables

In this method, we assume the solution to be the product of two functions, each of which involves only one of the variables. The following examples explain this method.

1. Variables Separable

If a differential equation can be written in the form

$$f(y)\,dy = \Phi(x)\,dx \tag{13}$$

We say that variables are separable, *y* on left hand side and *x* on right hand side. We get the solution by integrating both sides.

Working Rule:

Step1: Separate the variables as $f(y)dy = \Phi(x)dx$

Step 2: Integrate both sides as $\int f(y)dy = \int \Phi(x)dx$

Step 3: Add an arbitrary constant C on R.H.S.

2. Homogeneous Differential Equations

Working Rule:

Step 1: put y=vx so that $\frac{dy}{dx} = v + x\,\frac{dv}{dx}$

Step 2: Separate the variables.

Step 3: Integrate both the sides.

Step 4: Put $v = \frac{y}{x}$

3. Linear Differential Equations

A differential equation of the form

$$\frac{dy}{dx} + Py = Q \tag{14}$$

Working Rule:

Step 1: Convert the given equation to the standard form of linear differential equation

i.e, $\frac{dy}{dx} + Py = Q$

Step 2: Find the integrating factor i.e. I.F. = $e^{\int P\,dx}$

Step 3.Then the solution is $y\,(I.F.) = \int Q[\text{I.F.}]dx + C$

4. Equations Reducible To the Linear Form (Bernoulli Equation)

The equation of the form

$$\frac{dy}{dx} + Py = Qy^{n} \tag{15}$$

5. Exact Differential Equation

An exact differential equation is formed by directly differentiating its primitive (solution) without any other process

$$Mdx + Ndy = 0 \tag{16}$$

Step 1: Integrate M w.r.t. x keeping y constant.
Step 2: Integrate w.r.t. *y*, only those terms of *N* which do not contain *x*.
Step 3: Result of 1 + Result of 2 = Constant.

Complete Solution=Complementary Function + Particular Integral (17)

Let us consider a linear differential equation of the first order

$$\frac{dy}{dx} + Pq = Q \tag{18}$$

Its solution is $ye^{\int pdx} = \int\left(Qe^{\int pdx}\right)dx + C$

y = C.F. + P.I.

B. Method for Finding the Complementary Function

Step 1: In finding the complementary function, R.H.S. of the given equation is replaced by zero.

Step2: Let y = $C_1 e^{mx}$ be the C.F. of

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0 \tag{19}$$

Solution is

$$y.e^{-m_1 x} = \int\left(c_2 e^{m_1 x}\right)\left(e^{-m_1 x}\right)dx + c_1$$
$$= \int c_2 dx + c_1 = c_2 x + c_1$$
$$y = (c_2 x + c_1)\, e^{m_1 x}$$
$$C.F. = (c_1 + c_2 x)\, e^{m_1 x} \tag{20}$$

To Find the Value of $\frac{1}{f(D)} x^n \sin ax.$

$$\frac{1}{f(D)} x^n(\cos ax + i\sin ax) = \frac{1}{f(D)} x^n e^{ix} = e^{iax}\frac{1}{f(D+ia)} x^n$$
$$\frac{1}{f(D)} x^n \sin ax = \text{Imaginary part of } e^{iax}\frac{1}{f(D+ia)} x^n$$
$$\frac{1}{f(D)} x^n \cos ax = \text{Real part of } e^{ix}\frac{1}{f(D+ia)} x^n \tag{21}$$

General Method of Finding the Particular Integral of any Function $\emptyset$ (x) is:

$$P.I. = \frac{1}{D-a}\emptyset(x) = y \tag{22}$$

Cauchy Euler Homogeneous Linear Equations

$$a_n x_n \frac{d^n y}{dx^n} + a_{n-1} x^{n-1}\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_0 y = \emptyset(x) \tag{23}$$

Where a_0, a_1, a_3,………are constants, is called a homogeneous equation. or

$$x^2\frac{d^2y}{dx^2} = D(D-1)y \tag{24}$$

Similarity $x^3\frac{d^3y}{dx^3} = D(D-1)(D-2)y$

C. Method of Variation of Parameters

To find particular integral of

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = X \tag{25}$$

Working Rule:
Step 1: Find out the c.f i.e Ay_1+By_2
Step 2: Particular integral = uy_1+vy_2
Step 3: Find u and V by the formulae

$$u = \int \frac{-y_2 X}{y_1 y_2' - y_1' y_2}dx, \qquad v = \int \frac{y_1 X}{y_1 y_2' - y_1' y_2}dx \tag{26}$$

D. Simultaneous Differential Equations

If two or more dependent variables are functions of a single independent variable, the equations involving their derivatives are called simultaneous equations, e.g.

$$\frac{dx}{dt} + 4y = t$$
$$\frac{dy}{dt} + 2x = e^t \tag{27}$$

The method of solving these equations is based on the process of elimination, as we solve algebraic simultaneous equations.

IV. RESULTS OF EXPERIMENT

A. Application Of Differential Equation

1. Growth and Decay Problems

Let N (t) denotes the amount of substance (or population) that is either growing or decaying. If we assume that dN/dt, the time rate of change of this amount of substance, is proportional to the amount of substance present, then dN/dt = kn, or

$$\frac{dN}{dt} - KN = 0 \tag{28}$$

Where k is the constant of proportionality. We are assuming that N (t) is a differentiable, hence continuous, function of time. For population problems, where N (t) is actually discrete and integer-valued, this assumption is incorrect. Nonetheless, still provides a good approximation to the physical laws governing such a system.

Problem: A person places $20,000 in a savings account which pays *5* percent interest per annum, compounded continuously, Find (a) the amount in the account after three years, and (b) the time required for the account to double in value, presuming no withdrawals and no additional deposits.

Solution: Let N (t) denote the balance in the account at any time t. Initially, N (0) = 20,000. The balance in the account grows by the accumulated interest payments, which are proportional to the amount of money in the account. The constant of proportionality is the interest rate. In this case, k = 0.05 and equation (29) becomes

$$\frac{dN}{dt} - 0.05N = 0 \tag{29}$$

This differential equation is both linear and separable. Its solution is

$$N(t) = ce^{0.05t} \tag{30}$$

At $t = 0$, $N(0) = 20{,}000$, which when substituted into (31) yields $20{,}000 = ce^{0.05(0)} = c$. With this value of c, (31) becomes

$$N(t) = 20{,}000e^{0.05t} \tag{31}$$

Equation (31) gives the dollar balance in the account at any time t. substituting $t = 3$ into (31), we find the balance after three years to be

$$N(3) = 20{,}000e^{0.05(3)}$$
$$= 20{,}000(1.161834) \quad = \$23{,}236.68$$

We seek the time t at which N(t)= $40,000. Substituting these values into (31) and solving for t, we obtain

$$40{,}000 - 20{,}000e^{0.05t} \quad \rightarrow \quad 2 - e^{0.05t}$$

2. Vertical Motion

Problem: A particle falls under gravity in a resisting medium whose resistance varies with velocity. Find the relation between distance and velocity if initially the particle starts from rest.

Solution: By Newton's second law of motion, the equation of motion of the body is

$$m\frac{VdV}{dx} = mg - mkV \qquad \Rightarrow \qquad \frac{VdV}{dx} = g - kV$$
$$\frac{Vdv}{g - kV} = dx \qquad \Rightarrow \qquad -\frac{dV}{k} + \frac{g}{k}\frac{dV}{g - kV} = dx \tag{32}$$

Integrating, we get

$$-\frac{V}{k} + \frac{g}{k}\left(-\frac{1}{k}\right)\log(g - kV) = x + A$$
$$-\frac{V}{k} - \frac{g}{k^2}\log(g - kV) = x + A \tag{33}$$

Initially, $x = 0$, $V = 0$, $-\frac{g}{k^2}\log g = A$, Now (33) becomes

$$-\frac{V}{k} - \frac{g}{k^2}\log(g - kV) = x - \frac{g}{k^2}\log g$$
$$-\frac{V}{k} - \frac{g}{k^2}\log\left(\frac{g - kV}{g}\right) = x \tag{34}$$

3. Beam

A bar whose length is much greater than its cross-section and its thickness is called a beam.

Supported beam: If a beam may just rest on a support like a knife edge is called a supported beam.

Fixed beam: If one or both ends of a beam are firmly fixed then it is called fixed beam.

Cantilever: If one end of a beam is fixed and the other end is loaded, it is called a cantilever.

Bending of Beam: Let a beam be fixed at one end and the other end is loaded. Then the upper surface is elongated and therefore under tension and the lower surface is shortened so under compression.

Neutral Surface: In between the lower and upper surface there is a surface which is neither stretched nor compressed. It is known as a neutral surface.

Bending Moment: Whenever a beam is loaded it deflects from its original position. If M is the bending moment of the forces acting on it, then

$$M = \frac{EI}{R} \tag{35}$$

Where, E = Modulus of elasticity of the beam,
I = Moment of inertia of the cross-section of beam about neutral axis.
R = Radius of curvature of the curved beam

Thus equation (35) becomes

$$M = EI\frac{d^2y}{dx^2} \tag{36}$$

4. Boundary Conditions

1. End freely supported. At the freely supported end there will be no deflection and no bending moment.

$$y = 0, \quad \frac{d^2y}{dx^2} \tag{37}$$

2. Fixed end horizontally. Deflection and slope of the beam are zero.

$$y = 0, \quad \frac{dy}{dx} = 0 \tag{38}$$

3. Perfectly free end. At the free end there is no bending moment or shear force.

$$\frac{d^2y}{dx^2} = 0, \quad \frac{d^3y}{dx^3} = 0 \tag{39}$$

5. Convention of signs

The sign of the moment about NN′ on the left NN′ is positive if anticlockwise and negative if clockwise

The downward deflection is positive and length x on right-side is also positive. Slope $\frac{dy}{dx}$ is positive if downward.

Problem: The differential equation satisfied by a beam uniformly loaded (W kg/metre) with one end fixed and the second end subjected to tensile force P, is given by

$$E.I.\frac{d^2y}{dx^2} = Py - \frac{1}{2}Wx^2 \tag{40}$$

Show that the elastic curve for the beam with conditions $y = 0 = \frac{dy}{dx}$ at $x = 0$, is given by

$y = \frac{W}{Pn^2}(1 - \cosh nx) + \frac{Wx^2}{2P}$, where $n^2 = \frac{P}{EI}$

Solution:

We have, $$E.I\frac{d^2y}{dx^2} = Py - \frac{1}{2}Wx^2 \tag{41}$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{P}{E.I.}.y = -\frac{W}{2E.I.}x^2 \Rightarrow \left(D^2 - \frac{P}{E.I.}\right)y = -\frac{W}{2E.I.}x^2$$

A.E is

$$m^2 - \frac{P}{E.I.} = 0 \Rightarrow m^2 = \frac{P}{E.I.} = n^2 \Rightarrow m = \pm n$$

$$C.F. = c_1 e^{nx} + c_2 e^{-nx}$$

$$P.I. = \frac{1}{D^2 - \frac{P}{E.I}}\left(-\frac{W}{2.E.I.}\right)x^2 = -\frac{W}{2\,E.I.}\frac{1}{D^2 - n^2}x^2$$

$$\frac{W}{2n^2E.I.}\left(1 - \frac{D^2}{n^2}\right)^{-1}.x^2 = \frac{W}{2n^2E.I.}\left(1 + \frac{D^2}{n^2}\right).x^2 = \frac{W}{2n^2E.I.}\left(x^2 + \frac{2}{n^2}\right)$$

$$y = c_1 e^{nx} + c_2 e^{-nx} + \frac{W}{2n^2E.I.}\left(x^2 + \frac{2}{n^2}\right) \tag{42}$$

Differentiating (42) w.r.t. x, we get

$$\frac{dy}{dx} = nc_1e^{nx} - nc_2e^{-nx} + \frac{W}{2n^2E.I.}(2x) \tag{43}$$

Putting $x = 0, \frac{dy}{dx} = 0$ in (43), we get $0 = nc_1 - nc_2 \Rightarrow c_1 = c_2$

Putting $x = 0$, $y = 0$ in (42), we get

$$0 = c_1 + c_2 + \frac{W}{2n^2E.I.}\frac{2}{n^2} \Rightarrow 0 = c_1 + c_2 + \frac{W}{n^4E.I.} \tag{44}$$

Putting $c_1 = c_2$, in (44), we get

$$0 = 2c_1 + \frac{W}{n^4E.I.} \Rightarrow c_1 = -\frac{W}{2n^4E.I.},$$

Now $$n^2 = \frac{P}{E.I.} \Rightarrow n^2E.I. = P$$

$$c_1 = c_2 = \frac{W}{2n^2P}$$

Putting the values of c_1 and c_2 in (42), we get

$$y = \frac{-W}{2n^2P}(e^{nx} + e^{-nx}) + \frac{W}{2P}\left(x^2 + \frac{2}{n^2}\right)$$

$$y = \frac{-W}{n^2P}\cosh nx + \frac{W}{2P}x^2 + \frac{W}{Pn^2}$$

$$y = \frac{W}{Pn^2}(1 - \cosh nx) + \frac{Wx^2}{2P} \tag{45}$$

Problem: An infinitely long plane uniform plate is bounded by two parallel edges and an end at right angles to them. The breadth is π. This end is maintained at a temperature u_0 at all points and other edges are at zero temperature. Determine the temperature at any point of the plate in the steady state.

Solution: In the steady state, the temperature $u(x,y)$ at any point $P(x,y)$ satisfies aplace's equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0 \tag{46}$$

Under the boundary conditions:

$$u(0,y) = 0 \quad \forall y \tag{47}$$

$$u(\pi,y) = 0 \ \forall y \tag{48}$$

$$u(x,\infty)=0 \text{ in } 0 < x < \pi \tag{49}$$

$$u(x,0) = u_0 \ ; \qquad 0<x<\pi \tag{50}$$

Let us assume
Then the equation (46) reduces to

$$\ddot{X}\,Y + X\ddot{Y} = 0 \tag{51}$$

$$\Rightarrow \qquad \frac{\ddot{X}}{X} = -\frac{\ddot{Y}}{Y} = k \tag{52}$$

Where, *k* is separation parameter.

Case 1: When $k > 0$, Say $k=P^2$, where *p* is real. and

$$Y = C_3\cos py + C_4\sin py.$$

Hence

$$u(x,y) = (C_1e^{px}+C_2e^{-px})(C_3\cos py + C_4\sin py) \tag{53}$$

Case 2: When $k < 0$, Say $k=-p^2$, where *p* is real.

$$X = C_5\cos px + C_6\sin px$$

and $$Y = C_7e^{py}+C_8e^{-py}$$

Hence

$$u(x,y) = (C_5\cos px + C_6\sin px)\ (C_7e^{py}+C_8e^{-py}) \tag{54}$$

Case 3: When K=0, $X = C_9x + C_{10}$ and

$$X = C_{11}y + C_{12}$$

Hence

$$u(x,y) = (C_9x + C_{10})(C_{11}y + C_{12}) \tag{55}$$

The equation (53) and (55) do not satisfy the given conditions. The only possible appropriate solution is (54), i.e.,

$$u(0,y) = C_1(C_3e^{py}+C_4e^{-py}) = 0$$

$$\Rightarrow C_1 = 0 \quad \forall y$$

Hence

$$u(x,y) = C_2\sin px\,(C_3e^{py}+C_4e^{-py})$$

or $$u(x,y) = \sin px\,(C_3'e^{py}+C_4'e^{-py})$$

Again

$$u(\pi,y) = \sin p\pi\,(C_3'e^{py}+C_4'e^{-py}) = 0 \quad \forall y.$$

$$\sin p\pi = 0 = \sin n\pi$$

$$p = n$$

Again $u(x,\infty)=0 \quad \Rightarrow \quad C_3'=0$

Thus we have $u(x,y) = C_4'\sin nx.\ e^{-ny}$

$$u_n(x,y) = b_n\sin nx\ e^{-ny}$$

Hence the most general solution satisfying the boundary conditions is of the form

$$u(x,y) = \sum u_n(x,y) = \sum_{n=1}^{\infty} b_n\sin nx.\ e^{-ny} \tag{56}$$

Then $u(x,0) = u_0 = \sum_{n=1}^{\infty} b_n\sin nx$ (57)

Now $u_0 = \sum_{n=1}^{\infty} b_n\sin nx$

$$\Rightarrow b_n = \frac{2}{\pi}\int_0^{\pi} u_0\sin nx = \frac{2u_0}{n\pi}[1-(-1)^n]$$

$$=\begin{cases} 0, & n \text{ is odd} \\ \frac{4u_0}{n\pi}, & n \text{ is even} \end{cases}$$

Hence or

$$u(x,y) = \frac{4u_0}{\pi}\left[e^{-y}\sin x + \frac{1}{3}e^{-3y}\sin 3x + \frac{1}{5}e^{-5y}\sin 5x + \cdots\right] \tag{58}$$

which is the required solution. After considering a number of problems based on different types of differential equations, we found that the solutions of all differential equations satisfied the phenomena on theoretical basis and they belong to various category of differential equations described which are very useful in real life such as fluid engineering, biomedical, bioinformatics etc.

V. SUMMARY AND CONCLUSION

It has been observed that differential equations can describe any phenomena and the given conditions are completely solvable to find various results. In chapter 1 we are giving some introductory concept of differential equation and some basic concepts related to differential equation like degree order homogenous and non-homogenous differential equation with governing equation. In chapter 3 we are describing the complimentary function with some suitable example and the Cauchy equation, Laplace equation, heat equation and wave equation by using separation of variable and finally we are solving Laplace equation in polar form. In chapter 4 after considering number of problems based on different types of differential equations, we found that the solutions of all differential equations satisfied the phenomena of theoretical basis. The various category of differential equations described in the phenomena are genuine. The deflection u(x, y, t) of a rectangular membrane, square membrane and an infinitely long plane uniform plate is bounded by two parallel edges and an end at right angles to them etc. was found which showed a very good agreement with the physical results .In this chapter our first problem is growth and decay problems here we are taking this problem in only for the calculation of birth and death rate but it can be also easily implemented on the field of financial mathematics problem just like calculation of the profit and loss gain in any year by a business professional Similarly we are taking second problem dilution problems this problem was described a example of tank but it can be also used in chemical engineering problem regarding solvent, solute and dissolveness of any material, next we are implementing this problem in the field of electric circuit and computing different electric constants. our third problem is vertical motion is used in the projection of satellite, aerodynamics so we are explain it by simple example next we explain simple harmonic motion this is very useful in wave motion just like progressive wave, this phenomenon was not only arising in macroscopic also microscopic particle motion and finally we are describing projectile motion to describe this types of motion we are using Newtons law of motion and many times we are using vectors law.

We are also describing various features of BEAM and solving the governing equation by analytical method this is very useful for mechanical engineers. This differential equation are ordinary differential equation but we also describe Heat equation, wave equation and Laplace equation these equation are second order differential equation we are solving it by separation of variable and we are also deriving the governing equations such as Heat equation, wave equation and Laplace equation. Since we know that in case of ordinary differential equation we are taking only one independent variable but in case of partial differential equation we are taking more than one independent variable so partial differential equation provides more flexibility to design any Mathematical in comparison to ordinary differential equation. So now a days we are using very frequently partial differential equation but this is not meaning that ordinary differential equation have minimum scope both partial differential equation and ordinary differential equation have different entity and having different characteristic so to do any problem we have to know both ordinary as well as partial differential equation.

Future aspects:

In this thesis, we used analytical method for solving differential equation but many times we have seen that is not always possible to avoid these types of difficulties. We need numerical scheme but here we are using only analytical approach. In future we can extend our work and try to solve these problems by numerical method. Many times boundary condition was not smooth and standard. So in this type of situations, we cannot provide convergent solution just like drawing coating flow, burning of candle in these types of situations one boundary is first and other is varying very rapidly. This type of problems is known as moving boundary value problem.

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