

## Implementation of Asymptotic Capacity of Large Relay Networks for Cooperative Communication

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**Abstract:** This paper analyzes the capacity of a wireless relay network composed of a large number of nodes that operate in an amplify-and-forward mode and that divide into a fixed number of levels. The capacity computation relies on the study of products of large random matrices, whose limiting Eigen value distribution is computed via a set of recursive equations. Using free probability theory and assuming that the noise power at relays but not at destination is negligible, the closed-form expression of the asymptotic instantaneous end-to-end mutual information is derived as the number of antennas at all levels grows large. The so-obtained deterministic expression is independent from the channel realizations while depending only on channel statistics. This expression is also shown to be equal to the asymptotic average end-to-end mutual information. The singular vectors of the optimal pre-coding matrices, maximizing the average mutual information with finite number of antennas at all levels, are also obtained. It turns out that these vectors are aligned to the eigenvectors of the channel correlation matrices. Thus they can be determined using only the channel statistics. As the structure of the singular vectors of the optimal pre-coders is independent from the system size, it is also optimal in the asymptotic regime.

**Keywords:** Large Relay Networks, Conferencing, Asymptotic, Decode-and-Forward, Amplify-and-Forward.

### I. INTRODUCTION

Many cooperation strategies have been proposed in the literature based on different relaying techniques, such as amplify and forward (AF), decode and forward (DF) and coded cooperation, compress and forward (CF) etc. when these schemes are employed in a pair-wise cooperating system as shown in the below figure. We can assume that, at each instant in time, only one user acts as the source while the other user serves as the relay that forwards the source's message to the destination. The role between the source and the relay can be interchanged at any instant in time. If the DF scheme is employed the relay will decode and regenerate a new message to the destination in the subsequent time slot. At provide better detection performance. As an extension to the DF scheme, the message generated by the relay can be re-encoded to provide addition error protection, and such a scheme can also be referred to as coded cooperation. If the AF scheme is employed, the relay simply amplifies the received signal and forwards it directly to the destination without explicitly decoding the message. The SR scheme, on the other hand, is a dynamic scheme where relays are selected to retransmit the source message only if the relay path is sufficiently reliable.

This scheme can be applied on the top of both AF and DF schemes to improve cooperation efficiency. Among the many cooperation schemes proposed in the literature, DF, AF, and SR schemes are the most basic and widely adopted. More

sophisticated schemes, such as the CF scheme can also be devised by exploiting the statistical dependencies between the messages received at the relay and destination but require higher implementation complexity. Most cooperation strategies involve two phases of transmission: the coordination phase and the cooperative transmission phase. Coordination is especially required in cooperative transmission phase. Coordination is especially required in cooperative systems since the antennas are distributed among different terminals, as opposed to that in centralized MIMO systems. Although extra coordination may reduce bandwidth inefficiency, the cost is often compensated for by the large diversity gains experienced at high SNR. Specifically coordination can be achieved either by direct inter-user communication or by the use of feedback from the destination. Based on the information obtained through coordination, cooperating partners will compute and transmit messages so as to reduce the transmission cost or enhance the detection performance at the receiver. The rest of the paper is organized as follows. In Section II, we introduce the Methodology. In Section III, we discuss the Cooperation in relay channels. In Section IV, we present some simulation and numerical results. Finally, the paper is concluded in Section V.

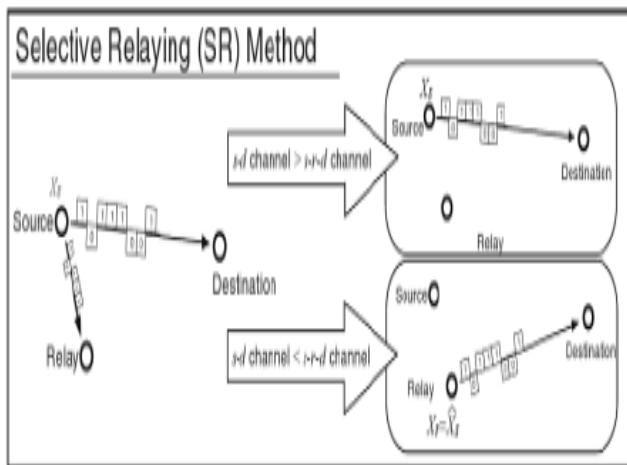
### II. METHODOLOGY

In this literature we introduce a conferencing link technique to increase the achievable rate of the system that is used for long transmission. For simplicity, the  $p$ -portion deterministic

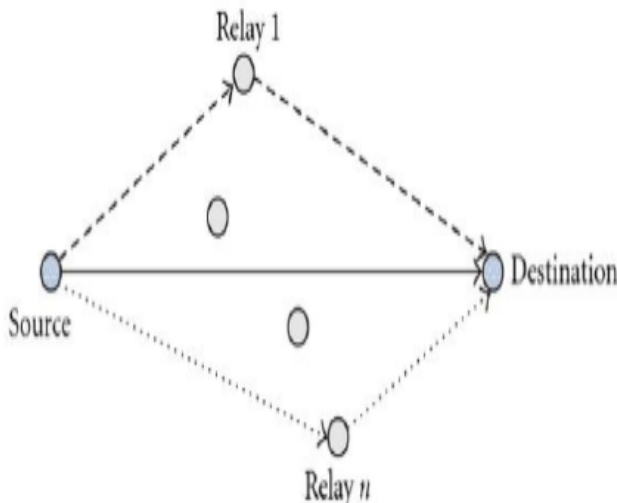
conferencing scheme is adopted here to provide a tractable achievable rate. In practical systems, it is costly to deploy MN conference links, which is exactly the reason why we propose a  $p$ -portion conferencing protocol to limit the percentage of conferencing connections.

**A. Cooperative Communication**

In cooperative wireless communication, we are concerned with a wireless network, of the cellular or ad hoc variety, where the wireless agents, which we call users, may increase their effective quality of service (measured at the physical layer by bit error rates, block error rates, or outage probability) via cooperation. We now review several of the main cooperative signaling methods. This method is perhaps closest to the idea of a traditional relay. In this method a user attempts to detect the partner's bits and then retransmits the detected bits (Fig. 1). The partners may be assigned mutually by the base station, or via some other technique. For the purposes of this tutorial we consider two users partnering with each other, but in reality the only important factor is that each user has a partner that provides a second (diversity) data path. The easiest way to visualize this is via pairs, but it is also possible to achieve the same effect via other partnership topologies that remove the strict constraint of pairing. Partner assignment is a rich topic whose details are beyond the scope of this introductory article.



**Fig.1. Selective Relaying Method.**



**Fig.2. Cooperative Relay Communication.**

**1. Amplify-And-Forward Methods**

Another simple cooperative signaling is the amplify-and-forward method. Each user in this method receives a noisy version of the signal transmitted by its partner. As the name implies, the user then amplifies and retransmits this noisy version. The base station combines the information sent by the user and partner, and makes a final decision on the transmitted bit (Fig. 1). Although noise is amplified by cooperation, the base station receives two independently faded versions of the signal and can make better decisions on the detection of information. In amplify-and-forward it is assumed that the base station knows the inter-user channel coefficients to do optimal decoding, so some mechanism of exchanging or estimating this information must be incorporated into any implementation. Another potential challenge is that sampling, amplifying, and retransmitting analog values is technologically nontrivial. Nevertheless, amplify-and-forward is a simple method that lends itself to analysis, and thus has been very useful in furthering our understanding of cooperative communication systems.

**2. Pre-coding**

Pre-coding is a generalization of beam forming to support multi-layer transmission in multi-antenna wireless communications. In conventional single-layer beam forming, the same signal is emitted from each of the transmit antennas with appropriate weighting such that the signal power is maximized at the receiver output. When the receiver has multiple antennas, single-layer beam forming cannot simultaneously maximize the signal level at all of the receive antennas. Thus, in order to maximize the throughput in multiple receive antenna systems, multi-layer beam forming is required. In point-to-point systems, pre-coding means that multiple data streams are emitted from the transmit antennas with independent and appropriate weightings such that the link throughput is maximized at the receiver output. In multi-user MIMO, the data streams are intended for different users (known as SDMA) and some measure of the total throughput (e.g., the sum performance) is maximized. In point-to-point systems, some of the benefits of pre-coding can be realized without requiring channel state information at the transmitter, while such information is essential to handle the co-user interference in multi-user systems.

**B. Pre-coding for Point-to-Point MIMO Systems**

In point-to-point multiple-input multiple-output (MIMO) systems, a transmitter equipped with multiple antennas communicates with a receiver that has multiple antennas. Most classic pre-coding results assume narrowband, slowly fading channels, meaning that the channel for a certain period of time can be described by a single channel matrix which does not change faster. In practice, such channels can be achieved, for example, through OFDM. The pre-coding strategy that maximizes the throughput, called channel capacity, depends on the channel state information available in the system.

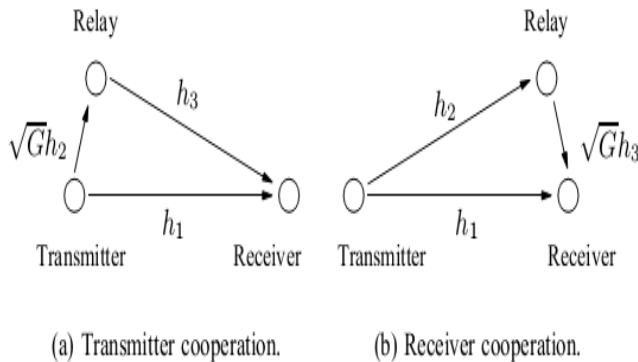
**III. COOPERATION IN RELAY CHANNELS**

**A. Transmitter vs. receiver cooperation**

We now consider a discrete-time memory less channel where a transmitter is communicating with a relay and

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receiver, as illustrated in Fig. 3. We assume a static channel, unit-variance AWGN, and a network average power constraint of  $P$ . The channel power gain between the cooperating nodes is  $G$ , while the other channels have unit magnitude:  $|h_i|=1, i=1, 2, 3$ .



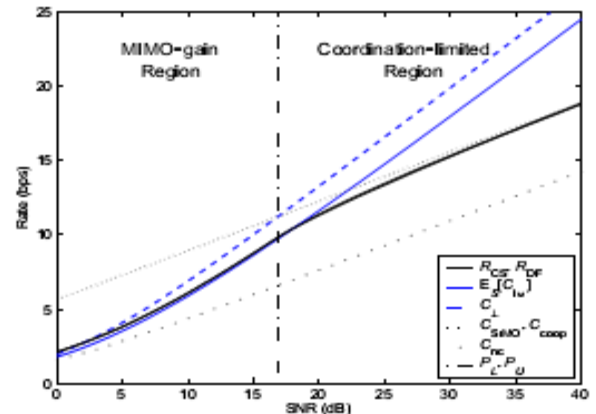
**Fig.3. Relay model**

The relay can be deployed near the transmitter or near the receiver and the questions is where the relay should be placed to maximize capacity between the transmitter and receiver. This question was investigated assuming both full and partial channel state information (CSI) as well as optimal or equal power allocation between the transmitter and relay. The capacity of the relay channel is unknown, but its capacity can be bounded using the cut-set upper bound and a lower bound based on any achievable transmission scheme. Two cooperation schemes were considered: the decode-and-forward scheme and the compress-and-forward scheme. In decode-and-forward transmission is done in blocks: the relay decodes the signal sent by the transmitter over one block, and in the subsequent block the relay and transmitter cooperatively send the message to the receiver. In compress and forward the relay sends a Wyner-Ziv compressed version of its received signal to the receiver. It was shown in that decode and forward (DF) is close to optimal when the relay is near the transmitter, and compress and forward (CF) is close to optimal when the relay is near the source.

Thus, in, DF was used for transmitter cooperation with the relay, and CF was used for receiver cooperation. Under these assumptions it was shown that when all nodes have equal average transmit power along with full channel state information (CSI), transmitter cooperation outperforms receiver cooperation, whereas the opposite is true when power is optimally allocated among the nodes but only receiver phase CSI is available. In addition, when the nodes have equal average power with receiver phase CSI only, cooperation is shown to offer no capacity improvement over a non-cooperative scheme with the same average network power. When the system is under optimal power allocation with full CSI, the decode-and-forward transmitter cooperation rate is close to its cut-set capacity upper bound, and outperforms compress-and-forward receiver cooperation. Moreover, it is shown that full CSI is essential in transmitter cooperation, while optimal power allocation is essential in receiver cooperation. These results were extended to Rayleigh fading channels in, where similar observations hold.

## B. Multiple-antenna relay vs. MIMO channel

When the relay has multiple antennas, we can compare the capacity of the cooperative system to that of a MIMO system. Consider the transmitter cooperation network in Fig. 3(a), where the transmitter, relay, and receiver has 1,  $M-1$ , and  $M$  antennas, respectively. The multiple-antenna relay channel performance represents an upper bound for the case where the transmitter utilizes multiple single-antenna nodes clustered together that coordinate to form a relay. While it is known that in the asymptotic regime, at a high SNR or with a large number of cooperating nodes, cooperative systems lack full multiplexing gain, in cooperative capacity gain was considered at moderate SNR with a fixed number of cooperating antennas. It was shown that up to a lower bound to an SNR threshold (MIMO-gain region), a cooperative system performs at least as well as a MIMO system with isotropic inputs; whereas beyond an upper bound to the SNR threshold (coordination-limited region), the cooperative system is limited by its coordination costs, and the capacity is strictly less than that of a MIMO orthogonal channel. The SNR threshold depends on the network geometry (the power gain  $G$  between the transmitter and relay) and the number of cooperating antennas  $M$ ; when the relay is close to the transmitter ( $G \gg 1$ ), the SNR threshold lower and upper bounds are approximately equal. As the cooperating nodes are closer, i.e., as  $G$  increases, the MIMO-gain region extends to a higher SNR. Whereas for a populous cluster, i.e., when  $M$  is large, the coordination-limited region sets in at a lower SNR.



**Fig.4. SNR regions of a 2x2 cooperative system**

For example, the cooperative capacity with  $M=2$  can be contrasted with that of a  $2 \times 2$  MIMO channel. The multiple-antenna relay channel cut-set bound  $R_{CS}$  and decode-and-forward rate  $R_{DF}$  are shown in Fig. 4, along with the SNR threshold lower and upper bounds  $P_L, P_U$ , the multiple-antenna capacity bounds, and for comparison the non-cooperative capacity  $C_{nc} = \log(1+2P)$ , which corresponds to the case where the relay is not used, and the source is under power constraint  $P$ . We assume the relay is near the source ( $G=100$ ); decode-and-forward is close to capacity-achieving as expected, and plots of  $R_{CS}, R_{DF}$  appear overlapped. The figure indicates that in the MIMO-gain region when  $P < P_L$ , the relay rate  $R_{DF}$  outperforms the isotropic-input MIMO capacity  $E_S [C_{IM}]$ . On the other hand, in the coordination-limited region, as  $P > P_U$ , the relay cut-set bound  $R_{CS}$  fails to parallel the orthogonal channel capacity  $C_{\perp}$ . Indeed, the

cooperative capacity is bottlenecked by the SIMO channel capacity  $C_{SIMO}$ , and which scales with the SNR as  $\Theta(\log P)$ , instead of  $\Theta(2 \log P)$ .

**C. Conferencing In relay channels**

We now consider a relay channel where the relay and receiver cooperate via orthogonal conference links with finite capacity. The conference cooperation model was introduced by Williams for a multiple-access channel (MAC) with conferencing encoders. By contrast, we consider conferencing between the relay and receiver. Specifically, we consider a discrete-time memory less channel where a transmitter is communicating with a relay and receiver, as illustrated in Fig. 5. We assume a static channel, unit-variance AWGN, perfect channel state information (CSI) everywhere, and an average total transmit power constraint  $P$ . The relative channel power gain between the relay and the receiver is  $g$ . We can assume real channel gains since the receivers can zero-phase the observed signals. The relay and receiver cooperate by way of a conference, as defined. The conference links are assumed to have finite capacity  $\alpha C$  and  $(1-\alpha) C$ , as shown in Fig. 5, where  $C$  is the total conference link capacity available between the receivers, and  $\alpha \in [0, 1]$  represents the allocation of conferencing resources in each direction. A conference is permissible if the total cardinality of the conference communications (possibly sent over multiple rounds) does not exceed that allowed by the capacity of the conference link.

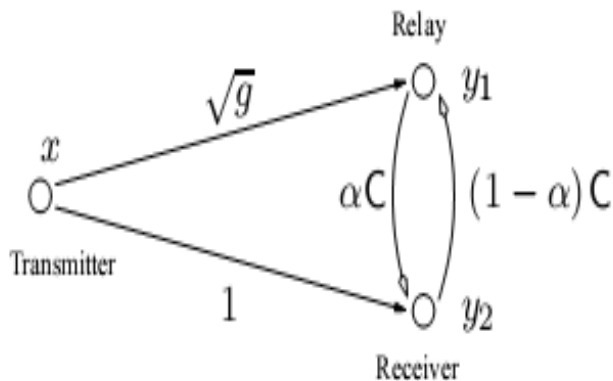
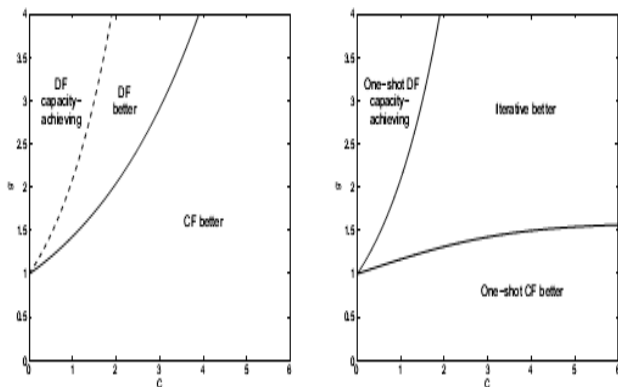


Fig.5. Conferencing relay and receiver.



(a) One-shot conferencing. (b) Iterative one-shot. Fig.6. The best cooperation strategy as a function of  $g$  and  $C$ .

Within this general conferencing setup the conference itself can be one-shot (non-iterative) or iterative. A comparison of these conferencing schemes was done in under different SNR and resource allocation assumptions. In particular, precise conditions that dictate which cooperation scheme achieves higher capacity were determined. It was shown that under one-shot conferencing, decode-and-forward (DF) is capacity-achieving when the relay has a strong channel relative to the conferencing capacity and power constraints. On the other hand, Wyner-Ziv compress-and-forward (CF) approaches the cut-set bound when the conference link capacity is large a plot of the conditions under which each cooperation scheme is better and if it is capacity-achieving is shown in Fig.6 (a). To contrast with one-shot conferencing, a two-round iterative conference, CF is done in the first round and DF in the second. A plot of the relative performance of one-shot versus a two-round iterative conference is shown in Fig. 6(b). The figure indicates that when the relay has a weak channel, the iterative scheme is disadvantageous. However, when the relay channel is strong, iterative cooperation, with optimal allocation of conferencing resources, outperforms one-shot cooperation provided that the conference link capacity is large.

**IV. SIMULATION RESULTS**

In this section, we present some simulation and numerical results to compare the performance among the proposed coding schemes. For simplicity, we assume that  $h_i$ 's and  $g_i$ 's are i.i.d. complex Gaussian random variable of  $CN(0, 1)$ ,  $|f_{i,k}| = 1$ ,  $P_s = 1$ ,  $P_r = 1$ , and  $N_0 = 1$ . The rates in all the simulations are averaged over 1000 fading realizations.

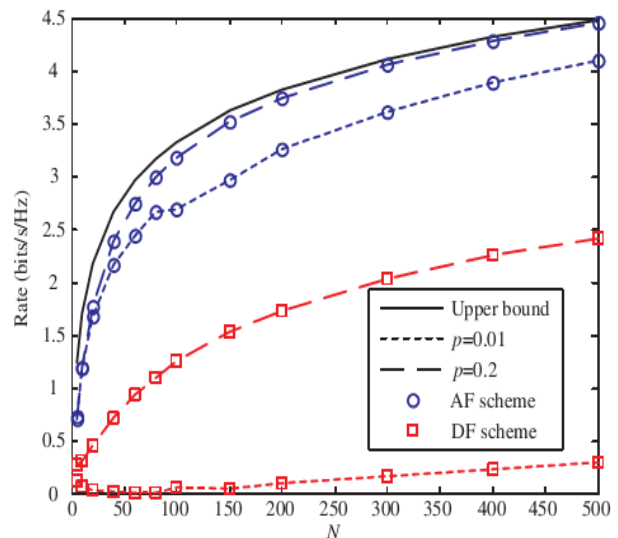
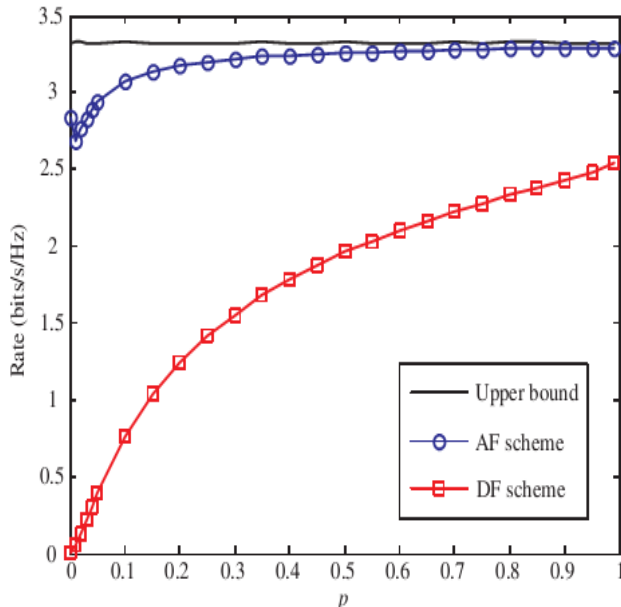


Fig.7. Achievable rates vs. the number of relays,  $P_s=1$ ,  $P_r=1$ ,  $P_c=1$ , and  $|f_{i,k}|=1$ .

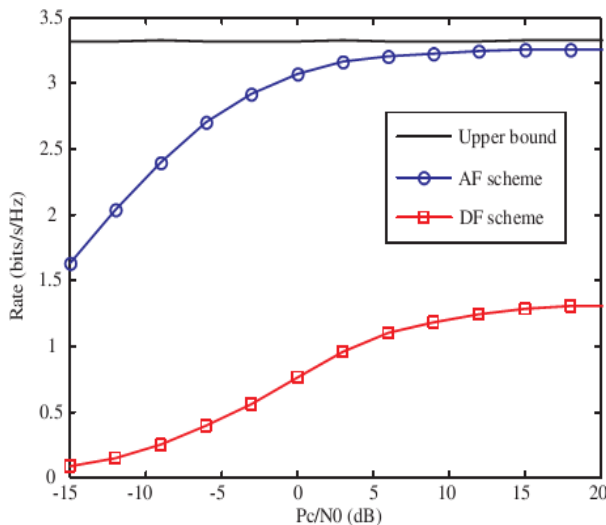
In Fig. 7, we show the capacity upper bound and the achievable rates for different  $p$  values, as the number of relays increases. For the AF relaying scheme, the gap between the upper bound and the achievable rate is very small for  $p=0.2$  and large  $N$  values. For the DF relaying scheme, when  $N$  is large, we observe that the DF rate and the capacity upper bound have the same scaling behavior. In Fig. 8, we plot the achievable rates as functions of  $p$ . For the AF relaying scheme, the  $p$  value does not need to be large to

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achieve most of the gains, i.e., around  $p=0.3$ ; on the other hand, conferencing may not strictly improve the AF rate: When  $p$  is close to zero, the achievable rate is lower than the case without relay conferencing, which is due to the sub-optimality of the combining scheme at the relays. For the DF relaying scheme, relay conferencing always helps, and there is a significant rate improvement as  $p$  increases.



**Fig.8. Achievable rates vs. the conferencing ratio,  $P_s=1$ ,  $P_r=1$ ,  $P_c=1$ ,  $|f_{i,k}|=1$ , and  $N=100$ .**



**Fig.9. Achievable rates vs. the conferencing link SNR,  $P_s=1$ ,  $P_r=1$ ,  $|f_{i,k}|=1$ ,  $N=100$ , and  $p=0.1$ .**

In Fig. 9, we plot the achievable rates as functions of the conferencing link SNR. It is observed that with medium-quality conferencing links (the SNRs of the conferencing links are around 5 dB), we achieve most of the gains introduced by relay conferencing for both the AF and DF relaying schemes.

## V. CONCLUSION

In this system, we investigated the achievable rate scaling laws of the DF and AF relaying schemes in a large Gaussian relay networks with conferencing links. By using this system we also prove the capacity of the system is increased. We

showed that for the DF relaying scheme, the rate scales as  $O(\log(N))$ , compared to  $O(\log(\log(N)))$  for the case without conferencing; for the AF relaying scheme, we proved that if the channel fading coefficients  $h_{i,k}$  and  $g_i$ 's are i.i.d., respectively, or  $N = M + 1$ , it asymptotically achieves the capacity upper bound as  $N$  goes to infinity.

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