



## Estimation of MIMO Channel Response for OFDM System with I/Q Imbalance and CFO

K.UPENDRA CHOWDARY<sup>1</sup>, G.RAMESH CHOWDARY<sup>2</sup>

<sup>1</sup> Asst Prof, ECE Dept, R.V.R&J.C.C.E, Guntur, AP-INDIA, Email: kurra.u@gmail.com.

<sup>2</sup> Asst Prof, ECE Dept, NIET, Hyderabad, AP-INDIA, Email: rameshchowdary7@gmail.com.

**Abstract:** In this project, we analyze the joint estimation of inphase and quadrature-phase (I/Q) imbalance and carrier frequency offset (CFO), and channel response for multiple-input multiple output (MIMO) orthogonal frequency division multiplexing (OFDM) systems using training sequences. A new concept called channel residual energy (CRE) is introduced. We show that by minimizing the CRE, we can jointly estimate the I/Q imbalance and CFO without knowing the channel response. The proposed method needs only one OFDM block for training and the training symbols can be arbitrary. Moreover when the training block consists of two repeated sequences, a low complexity two-step approach is proposed to solve the joint estimation problem. Simulation results show that the mean-squared error (MSE) of the proposed method is close to the Cramer-Rao bound (CRB).

**Keywords:** OFDM, MIMO, I/Q Imbalance, CFO, Chan-nel Estimation.

### I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has become the standard of choice for wireless LAN's such as IEEE 802.11a, and is being considered for several IEEE 802.11 and 802.16 standards. The popularity of OFDM arises from the balanced transceiver complexity, and the time-frequency granularity that it offers. However, synchronization continues to be a critical challenge.

This work focuses on carrier frequency offset (CFO) synchronization, assuming perfect frame and timing synchronization. Data-aided CFO estimation in current OFDM systems employs a preamble made of a number, say  $J$ , of repetitive slots (RS). This preamble is obtained using one OFDM symbol after deactivating all sub carriers except those whose frequencies are integer multiples of  $J$ . It has been shown that the RS-based CFO maximum likelihood (ML) estimator is identical to the null-sub carrier(NSC)-based ML estimator in the absence of virtual sub carriers, which are the sub carriers at the edges of the allocated frequency band that are deactivated in order to avoid interference with adjacent systems.

This addresses the issue of optimal preamble design using the Cramer-Rao bound (CRB) as a metric. This involves optimizing  $J$  and the power loading. The optimal value of  $J$  is a trade-off between the multipath diversity gain and the number of unknowns to be

estimated. In the case of uncorrelated channel taps, uniform power loading is optimal. In the case of correlated channel taps, that uniform power loading of the active sub carriers is no longer optimal and better power loading schemes are proposed.

### II. MULTI INPUT MULTI OUTPUT-OFDM

Multiple Input, Multiple Output Orthogonal Frequency Division Multiplexing is a technology developed by Iospan Wireless that uses multiple antennas to transmit and receive radio signals. MIMO-OFDM will allow service providers to deploy a Broadband Wireless Access (BWA) system that has Non-Line-of-Sight (NLOS) functionality. Specifically, MIMO-OFDM takes advantage of the multipath properties of environments using base station antennas that do not have LOS. According to Iospan,

"In this environment, radio signals bounce off buildings, trees and other objects as they travel between the two antennas. This bouncing effect produces multiple "echoes" or "images" of the signal. As a result, the original signal and the individual echoes each arrive at the receiver antenna at slightly different times causing the echoes to interfere with one another thus degrading signal quality. The MIMO system uses multiple antennas to simultaneously transmit data, in small pieces to the receiver, which can process the data flows

and put them back together. This process, called spatial multiplexing, proportionally boosts the data-transmission speed by a factor equal to the number of transmitting antennas. In addition, since all data is transmitted both in the same frequency band and with separate spatial signatures, this technique utilizes spectrum very efficiently.

There exist several communication transmission models as follows (see Fig.1):

1. Single-input-and-single-output (SISO) system: It uses only one antenna both at the transmitter and receiver.
2. Single-input-and-multiple-output (SIMO) system: It uses a single transmitting antenna and multiple receiving antennas [3].
3. Multiple-input-and-single-output (MISO) system: It has multiple transmitting antennas and one receiving antenna.
4. Multiple-input-multiple-output (MIMO) system: It uses multiple antennas both for transmission and reception. Multiple transmitting and receiving antennas will achieve antenna diversity without reducing the spectral efficiency.

In MIMO system, a number of antennas are placed at the transmitting and receiving ends, their distances are separated far enough. The distance between different base station antennas can be set as 10 times the carrier wavelength and mobile station antennas can be separated by half carrier wavelength. The idea is to realize spatial multiplexing and data pipes by developing space dimensions which are created by multi-transmitting and receiving antennas. The block diagram in Fig.1 illustrates the antenna configuration is space-time systems

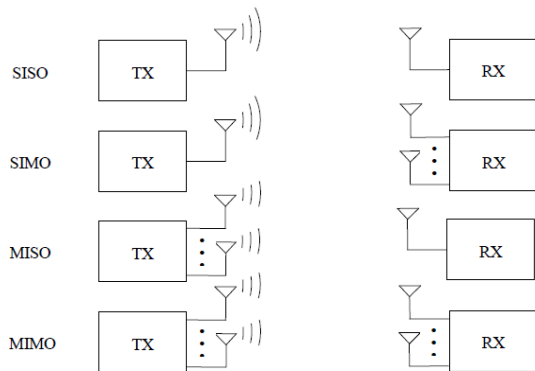


Fig:1 Basic Structure of MIMO System

### III. MIMO OFDM SYSTEM MODEL

#### A. ASSUMPTIONS IN THIS PROJECT

- Usage of cyclic Prefix
- Impulse response of the channel shorter than Cyclic Prefix
- Slow fading effects so that the channel is time-invariant over the symbol interval
- Rectangular Windowing of the transmitted pulses
- Perfect Synchronization of transmitter and receiver
- Additive, white, Gaussian channel noise

#### B. MODULE DESCRIPTION:

1. Estimate the MIMO channel
2. Joint Estimation of Channel Response and I/Q Imbalance

#### B1: Estimate the MIMO channel

In that module there are no CFO and I/Q imbalance. So for the purpose of channel estimation, we rewrite the received signal as

$$R = HX + q \tag{1}$$

Where  $H$  = complex channel gain

$X$  = transmission signal

$q$  = channel noise

$R$  = received signal

Given  $s_i$ , or equivalently  $x_i$  (see Fig. 1), we want to estimate the channel response from the received vector. Since it has been shown [6] that the channel responses can be estimated separately for each receive antenna, we drop the receive antenna index  $k$  in the following discussion for simplicity. Suppose that there are no CFO and I/Q imbalance. For the purpose of channel estimation, we rewrite the received signal in as

$$r = [X_0 \ X_1 \ \dots \ X_{N_t-1}] \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{N_t-1} \end{bmatrix} + a_i \tag{2}$$

where  $\mathbf{h}$  is the  $M \times 1$  vector defined in (2) and  $\mathbf{X}$  is an  $M \times M$  circulant matrix with the first column  $X_j$ .

## Estimation of MIMO Channel Response for OFDM System with I/Q Imbalance and CFO

Define the  $M \times L$  sub matrix  $A_j$  which consists of the first  $L$  columns of  $X_j$  and let  $c_j$  be

$$c_j \cong [h_j(0) \ h_j(1) \ \dots \dots \dots \ h_j(L-1)]^T \quad (3)$$

Define

$$c \cong [c_0^T \ c_1^T \ \dots \ c_{N_t-1}^T]^T \quad (4)$$

and

$$A \cong [A_0 \ A_1 \ \dots \ A_{N_t-1}] \quad (5)$$

then

$$r = Ac + q \quad (6)$$

From above eqn, we see that the  $L_{N_t \times 1}$  channel vector  $c$  is identifiable if and only if the  $L_{N_t}$  matrix  $A$  has full column rank. Thus a necessary condition for channel identifiability is  $L \geq L_{N_t}$ . Assume  $q$  is a complex Gaussian random vector with covariance matrix  $R_q$ . Then a least-squares estimate of  $c$  is given by

$$\hat{c} = (A^+A)^{-1}A^+r \quad (7)$$

Define the error vector as  $e$ . The design that minimizes the mean squared error  $E\{\|e\|^2\}$ . It was shown that the optimal training sequences from different transmit antennas must satisfy. The general closed-form solution for  $x_i$  that satisfies the orthogonality condition. When the training sequences are orthogonal, the least-squares estimate of channel response becomes

$$\hat{c} = A^+r \quad (8)$$

In the following, we will describe the above estimation method using a different formulation. This formulation is needed later for the joint estimation of I/Q imbalance, CFO and channel response. Define

$$\rho = \frac{M}{N_t} \quad (9)$$

In this paper, we assume that  $M$  is a multiple of  $N_t$  and the proposed method can be extended to the case where  $M$  is not a multiple of  $N_t$  by simple modification. Furthermore, we assume that

$$\rho \geq L \quad (10)$$

so that the channel identifiability condition  $M \geq N_t L$  is guaranteed. Suppose we append  $(\rho-L)$  zeros to the length- $L$  vectors  $c_i$  to obtain the  $\rho \times 1$  vectors

$$d_i \cong \begin{bmatrix} c_i \\ 0 \end{bmatrix}, \text{ for } i = 0, 1, \dots, N_t - 1. \quad (11)$$

Collecting all the vectors  $d_i$  for  $0 \leq i \leq N_t - 1$ , we form the  $M \times 1$  vector

$$d \cong [d_0^T \ d_1^T \ d_2^T \ \dots \ d_{N_t-1}^T]^T \quad (12)$$

Let  $A'k$  be any  $M \times (\rho - L)$  matrix such that the following  $M \times M$  matrix  $B$  is invertible

$$B \cong [A_0 A'_0 \ A_1 A'_1 \ \dots \ A_{N_t-1} A'_{N_t-1}] \quad (13)$$

The vector  $r$  in can be rewritten as  $r = Bd + q$ . Since  $B$  is  $M \times M$ , the estimate of MIMO Channel response is given by

$$\hat{d} = [\hat{d}_0^T \ \hat{d}_1^T \ \dots \ \hat{d}_{N_t-1}^T]^T = B^{-1}r \quad (14)$$

The estimated channel response  $\hat{c}_j$  is given by the first  $L$  entries of  $\hat{d}_j$ . When the training sequences are orthogonal as in with  $\epsilon_s = 1$ , the columns in  $A_k$  are orthonormal. In this case, one can choose the matrices  $A'_k$  in such that  $B$  is unitary. The channel estimate is thus given by  $\hat{d} = B^T r$ . If in addition, the orthogonal sequences are the optimal sequences proposed in,  $B$  can be chosen to be unitary and circulant. The unitary and circulant properties of  $B$  is shown below. In the sequences for the  $i$ th transmit antenna are given by

$$s_i(k) = s_0(k) e^{-j\frac{2\pi}{M}\rho k} \quad (15)$$

where  $s_0(k)$  are constant modulus modulation symbols transmitted through the  $0^{th}$  transmit antenna. Define two  $M \times M$  diagonal matrices

$$S_0 \cong \text{diag} [s_0(0) \ s_0(1) \ \dots \ s_0(M-1)] \quad (16)$$

$$\Delta_i \cong \text{diag} \left[ 1 \ e^{-j\frac{2\pi}{M}\rho k} \ \dots \ e^{-j\frac{2\pi}{M}(M-1)\rho k} \right] \quad (17)$$

The  $M \times M$  circulant matrix  $X$  is given by

$$X_i = W^T S_0 \Delta_i W \quad (18)$$

Denote the  $k$ th column of the DFT matrix  $W$  as  $w_k$ ,

$$\Delta_i w_k = w((k + i\rho)_M) \quad (19)$$

we see that the  $k$ th column of  $X_0$  is equal to  $W^T S_0 w_k$  (where  $S_0 = I$ ) and the  $k$ th column of  $X_i$  is equal to  $W^T S_0 (\Delta_i w_k)$ . we conclude that the columns of  $X_i$  are those of  $X_0$  circularly shifted by  $i\rho$  columns. That is,

$$[X_{0,i} \ X_{1,i} \ \dots \ X_{M-1,i}] = [X_{((\rho))M,0} \ X_{((\rho+1))M,0} \ \dots \ X_{((\rho-1))M,0}] \quad (20)$$

where  $X_{k,i}$  is the  $k$ th column of  $X_i$  and  $X_{((k))M,0}$  is the column of  $X_0$ . Recall that  $A_i$  are formed by the first  $L$  columns of  $X_i$ . If the  $M \times (\rho - L)$  matrix  $A'$  is defined such that contains the first columns of  $X_i$ , we see that the  $M \times M$  matrix  $B$  defined is in fact equal to  $X_0$ , which is unitary and circulant.

**B2: Joint Estimation of Channel Response and I/Q Imbalance:**

In this module, we assume that there is no CFO. Hence we have  $\alpha = 0$  and  $E = I$ . and  $\mu r$  is related to the received vector  $z$  as

$$\mu r = \frac{z - \alpha z^*}{1 - |\alpha|^2} \quad (21)$$

Where  $\alpha = I/Q$  mismatch parameter,

if  $\alpha$  is given, an estimate of the MIMO channel response can be obtained as

$$\mu \hat{d} = \mu [\hat{d}_0^T \ \hat{d}_1^T \ \dots \ \hat{d}_{N_t-1}^T]^T \quad (22)$$

$$\mu \hat{d} = B^{-1} \mu r = B^{-1} \frac{z - \alpha z^*}{1 - |\alpha|^2} \quad (23)$$

When  $B$  is  $\alpha$  estimated perfectly, the first  $L$  entries of each  $\hat{h}_j$  in the above expression will give us an estimate of the channel response and the last  $(\rho - L)$  entries of  $\hat{d}_j$  are solely due to the channel noise. For moderately high SNR, the energy of these entries should be small. Let us define a quantity called the channel residual energy (CRE) as

$$CRE \cong \sum_{i=0}^{N_t-1} \sum_{l=L}^{\rho-1} |[\mu \hat{d}_i]_l|^2 \quad (24)$$

where  $[\mu \hat{d}_i]$  denotes the  $i$ th entry of  $\mu \hat{d}_i$ . Any error in the estimation of  $\alpha$  will increase the CRE (see the analysis at the end of this section). Based on this observation, by minimizing the CRE we are able to estimate the I/Q parameter  $\alpha$  without knowing the channel response. Our goal is to find  $\alpha$  that minimizes the CRE. Since for most applications,  $\alpha$  is small, can be approximated as

$$CRE \approx ||PB^{-1}(z - \alpha z^*)||^2 \quad (25)$$

From linear algebra, it is known that the optimal  $\alpha$  that/minimizes the CRE is

$$\alpha_{opt} = \frac{(PB^{-1}z^*)^T (PB^{-1}z)}{|PB^{-1}z^*|^2} \quad (26)$$

By substituting  $\alpha_{opt}$  and get the estimated MIMO channel response  $\mu \hat{d}_i$ . For the compensation of I/Q imbalance, one can employ to obtain  $\mu$ . Notice that there is no need to compensate the factor  $\mu$  because it will be canceled when we use to implement the FEQ. From that to get  $\alpha_{opt}$ , we only need to compute  $B^{-1}z$  (as  $B$  is fixed,  $B^{-1}$  can be precomputed) and perform vector inner products at the numerator and denominator. When the training sequence is used,  $B$  becomes unitary and circulant. As  $B^{-1}$  is also circulant and unitary,  $B^{-1}z$  can be efficiently realized using circular convolution.

**IV. SIMULATION RESULTS**

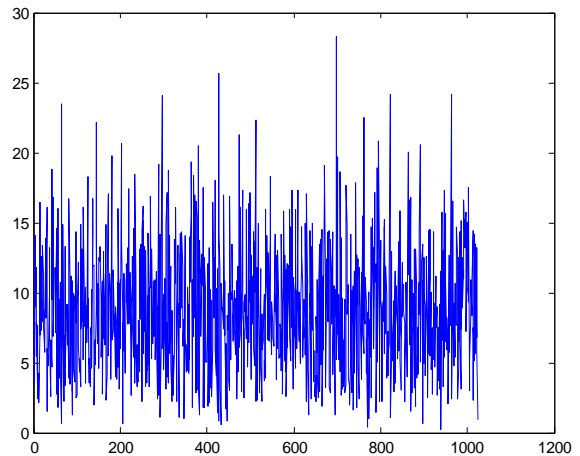


Fig1. Generated AWGN in Frequency Domain.

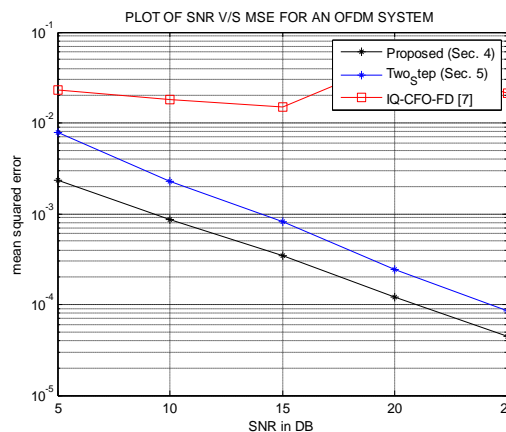


Fig2. Plot of Mean Square Error in a normal OFDM System.

# Estimation of MIMO Channel Response for OFDM System with I/Q Imbalance and CFO

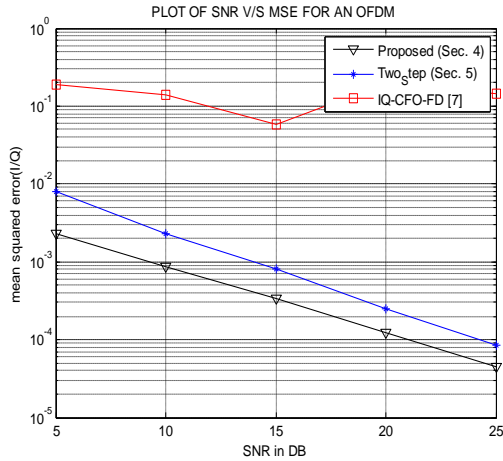


Fig3. Plot of Mean Square Error for I/Q

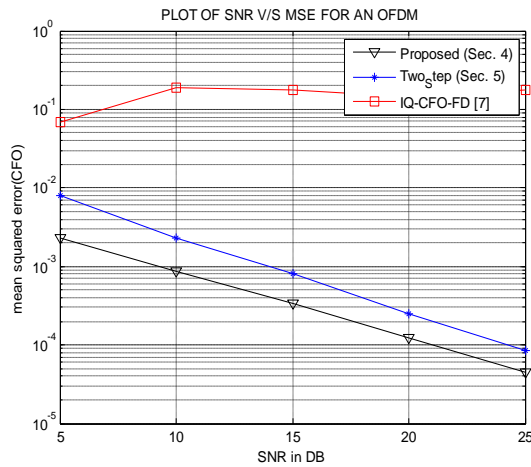


Fig4. Plot of Mean Square Error of CFO

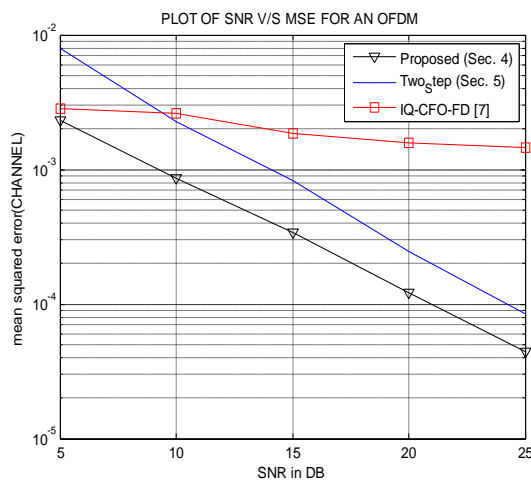


Fig5. Plot MSE for Channel Estimation with I/Q and CFO

## V. CONCLUSION

we propose new methods for the joint estimation of the I/Q imbalance, CFO and channel response for MIMO OFDM systems by using training sequences. When only one OFDM block is available for training, the first method is able to give an accurate estimate of the CFO, I/Q parameter and channel responses.

## VI. REFERENCES

1. H. Chung and S.-M. Phoong, "Joint estimation of I/Q imbalance and channel response for MIMO OFDM system," in Proc. EUSIPCO, Sept. 2007.
2. K. Lee and J. Chun, "Frequency-offset estimation for MIMO and OFDM systems using orthogonal training sequences," IEEE Trans. Veh. Technol., Jan. 2007.
3. Barhumi, G. Leus, and M. Moonen, "Optimal training design for MIMO OFDM systems in mobile wireless channels," IEEE Trans. Signal Process., June 2003.
4. Barhumi, G. Leus, and M. Moonen, "Optimal training design for MIMO OFDM systems in mobile wireless channels," IEEE Trans. Signal Process., June 2003.
5. Tarighat, A. H. Sayed, "Compensation schemes and performance analysis of I/Q imbalances in OFDM receivers," IEEE Trans. Signal Process., Aug. 2005.
6. M. Windisch and G. Fettweis, "Preamble design for an efficient I/Q imbalance compensation in OFDM direction-conversion receivers," in Proc. 10th Intl. OFDM Workshop, Aug./Sep.2005.
- [6] T. M. Schmidl and D. C. Cox, "Robust frequency and timing synchronization for OFDM," IEEE Trans. Commun., vol. 45, no. 12, pp. 1613-1621, Dec. 1997.
- [7] J. Tubbax, A. Fort, L. Van der Perre, S. Donnay, M. Engels, M. Moonen, and H. De Man, "Joint compensation of I/Q imbalance and frequency offset in OFDM systems," in Proc. IEEE Globecom, 2003.
- [8] M. Morelli and U. Mengali, "Carrier-frequency estimation for transmissions over selective channels," IEEE Trans. Commun., Sep. 2000.
- [9] Y. Li, "Simplified channel estimation for OFDM systems with multiple transmit antennas," IEEE Trans. Wireless Commun., vol. 1, no. 1, pp. 67-75, Jan. 2002.
- [10] I. Barhumi, G. Leus, and M. Moonen, "Optimal training design for MIMO OFDM systems in mobile wireless channels," IEEE Trans. Signal Process., June 2003.