



A Low-Complexity Optimum Detection Based On High-Rate Full-Diversity MIMO Space Time Block Codes

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Abstract: The 2×2 MIMO profiles included in Mobile Wi-MAX specifications are Alamouti's space-time block code (STBC) for transmit diversity and spatial multiplexing (SM). The former has full diversity and the latter has full rate, but neither of them has both of these desired features. An alternative 2×2 STBC, which is both full rate and full diversity, is the Golden code. It is the best known 2×2 STBC, but it has a high decoding complexity. Recently, the attention was turned to the decoder complexity, this issue was included in the STBC design criteria, and different STBCs were proposed. In this paper, we first present a high-rate full-diversity 2×2 STBC design leading to substantially lower complexity of the optimum detector compared to the Golden code with only a slight performance loss. We provide the general optimized form of this STBC and show that this scheme achieves the diversity multiplexing frontier for square QAM signal constellations. Then, we present a variant of the proposed STBC, which provides a further decrease in the detection complexity with a rate reduction of 25% and show that this provides an interesting trade-off between the Alamouti scheme and SM.

Keywords: ML detection, multiple-input multiple-output (MIMO), space-time block codes (STBCs).

I. INTRODUCTION

Multiple input multiple-output (MIMO) techniques based on using multiple antennas at transmitter and receiver can provide spatial diversity, multiplexing gain, interference suppression, and make various tradeoffs between them. These techniques have been incorporated in all of the recently developed wireless communications system specifications including the IEEE 802.16e-2005 standard [1] for mobile broadband wireless access systems. From the MIMO schemes included in the IEEE 802.16e specifications, the WiMAX Forum has specified two mandatory profiles for use on the downlink.

One of them is based on the space-time block code (STBC) proposed by Alamouti for transmit diversity [2]. This code achieves a diversity order that is equal to twice the number of antennas at the receiver, but it is only half-rate. (In this paper, the rate is defined as the number of transmitted symbols per antenna use.) The other profile is spatial multiplexing (SM), which uses two transmit antennas to transmit two independent data streams. This scheme is

full-rate, but it does not benefit from any diversity gain at the transmitter.

This code is a variant of the Golden code [3], which is known to be one of the best 2×2 STBCs achieving the diversity multiplexing frontier [4]. But the problem of this code is its detection complexity, which grows as the fourth-power of the signal constellation size, and this makes it impractical for low-cost wireless user terminals. Recently, motivated by the orthogonality of the Alamouti scheme, new high rate (full-rate) full-diversity (FR-FD) 2×2 STBCs were proposed independently in [5], [8]–[10]. These codes achieve the diversity-multiplexing frontier with reduced detection complexity.

In this paper, we describe the STBC originally proposed in [5], discuss its basic properties, and compare it with the best known STBC to date. We provide the general optimized form of this STBC whose optimum detection complexity (using exhaustive search) grows at most quadratic ally with the size of the signal constellation and show that this scheme achieves the diversity-multiplexing

frontier for square QAM signal constellations. We also present a rate-3/4 variant of this STBC which provides an interesting trade-off between the Alamouti scheme and SM.

The rest of the paper is organized as follows. First, in Section II, we briefly discuss the general design criteria for STBCs. Sections III and IV are devoted to the proposed 2x2 STBCs and the relevant comparisons. Specifically, we first describe the proposed scheme and compare its features with the best known alternatives. Then, we describe the corresponding maximum likelihood (ML) detector including both exhaustive search and sphere decoder (SD), and analyze the optimized form of the proposed STBC. In Section IV, we present the rate-3/4 STBC with a further reduction in receiver complexity. Finally, we present some numerical comparisons in Section V, and we give our conclusions in Section VI.

II. STBC Design Criteria

A. Pair-wise Error Probability Analysis

B.

We consider that the transmitter does not have any channel state information while the receiver knows the channel perfectly. For 2 x 2 MIMO transmission, we write

$$Y = HX + Z \quad (1)$$

Where H is the 2 x 2 channel matrix with the entries of complex channel gains, X is the 2 x 2 codeword matrix

$$X = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} \quad (2)$$

Whose elements take values from the codebook X, Y includes the received signals and Z denotes the matrix of additive circularly symmetric complex Gaussian noise samples with spectral density N0, respectively.

Recently proposed STBC schemes mainly rely on analysis of the pair-wise error probability (PEP) P(X → X̂) which is the probability that X̂ is detected while X is transmitted. If the difference matrix (X - X̂) is full rank for all codeword pairs, then the code is said to have full diversity. For high SNR values, the most important parameter is the diversity gain, which dominates the steepness of the bit-error rate (BER) curve. After ensuring full-diversity, we need to maximize the coding gain which can be defined for a 2 x 2 STBC as

$$\delta(\mathcal{X}) = \min_{\substack{X, \hat{X} \in \mathcal{X} \\ X \neq \hat{X}}} |\det(X - \hat{X})|^2. \quad (3)$$

B. Detection complexity

In the design of STBCs another important criterion is the decoding complexity. This is highly crucial especially for mobile applications. The Golden code is the best known full-rate 2 x 2 STBC which satisfies the rank criterion with

a high coding gain. Therefore, other FR-FD STBCs should be introduced as alternatives to the Golden code which have lower optimum decoding complexity. The results available in the literature suggest that there is an intrinsic tradeoff between error performance and detection complexity. The Golden code, which we denote by in the sequel, provides FR-FD with a coding gain of 16/5 and achieves substantially better performance than SM whose diversity order is limited to the number of receive antennas. But, as explained above, this code has an inherent detection complexity problem.

For full-rate 2 x 2 STBCs, the optimum receiver evaluates the ML function expressed as:

$$D(s_1, s_2, s_3, s_4) = \|Y - HX\|^2 \quad (4)$$

The norm given in (4) is actually the squared Euclidean distance between the received noisy signal and the noiseless signal corresponding to that quadruplet. For a signal constellation with M points, this receiver involves the computation of M⁴ Euclidean distances and selects the symbol quadruplet minimizing this distance. This complexity is of course prohibitive in practical applications with the 16-QAM and 64-QAM signal constellations and current state of technology. One possible solution is to use SD whose performance and complexity are upper bounded by those of ML detection based on exhaustive search. However, even the use of SD would require a high number of computations for satisfactory detection performance.

III. Proposed STBC and Comparison with Existing FR-FD 2x2 STBCs

Now we turn our attention to the recently proposed FR-FD 2 x 2 STBC schemes. They attempt to maximize both the diversity gain and the coding gain, while leading to an optimum detection of reduced complexity. More specifically, these schemes are FR-FD 2x2 STBCs whose optimum receiver has a complexity that is only proportional to M². Comparing their complexity to that associated to Xg, it becomes clear that these codes make the implementation of FR-FD 2 x 2 STBCs with an optimum receiver more realistic.

The STBC presented in [8] is a combination of the original Alamouti scheme and a recoded scheme having also an Alamouti structure. In contrast, our STBC directly combines two Alamouti schemes and evenly distributes the average transmitted energy for each symbol per channel use. Since the transmitted signal is a combination of two symbols only, it has a lower peak-to-average power ratio (PAPR) than the STBCs presented in [8][10] whose components are sums of more than two signals.

In this code, a group of 4 symbols (s1, s2, s3, s4) is transmitted as follows:

$$X_{new} = \begin{bmatrix} as_1 + bs_3 & -cs_2^* - ds_4^* \\ as_2 + bs_4 & cs_1^* + ds_3^* \end{bmatrix} \quad (5)$$

A careful look clearly shows that (5) is nothing but a simple linear combination of two Alamouti schemes. Here, a, b, c, and d are complex-valued design parameters. They are chosen such that the resulting STBC attains FR-FD transmission in a quasi static Rayleigh fading channel. Here, the first condition ensures the transmission of equal average power at each symbol time, while the second condition ensures that equal average total power is transmitted for each symbol. Before giving the details related to the optimization of the design parameters we will give a brief comparison among the existing STBCs and explain the reduced complexity detection capability of the presented STBC.

A. Comparison with the Existing STBCs

Similar to the other STBCs mentioned above, the proposed STBC X_{new} falls into the class of linear dispersion codes [12] which can be written in the form

$$X = \sum_{k=1}^4 (s_{k,R} A_k + j s_{k,I} B_k) \tag{6}$$

Where $s_{k,R}$ and $s_{k,I}$ denote the real and imaginary parts of the symbol s_k , respectively, and $A_k, B_k, k = 1, \dots, 4$, are 2×2 complex-valued weight matrices of X .

Now, in order to make a more detailed comparison, we use vector representation and introduce the following notation. First, define the column vectors

$$\begin{aligned} \mathbf{x}^- &= [x_{11}, x_{21}, x_{12}, x_{22}]^T, \\ \mathbf{y}^- &= [y_{11}, y_{21}, y_{12}, y_{22}]^T \text{ and} \\ \mathbf{z}^- &= [z_{11}, z_{21}, z_{12}, z_{22}]^T, \end{aligned}$$

Which are obtained by stacking the columns of the matrices X, Y and Z , respectively, one after the other. Next, we define the corresponding real-valued column vector as

$$\bar{\mathbf{x}}_R = [\Re\{x_{11}\}, \Im\{x_{11}\}, \Re\{x_{21}\}, \Im\{x_{21}\}, \dots, \Re\{x_{22}\}, \Im\{x_{22}\}]^T. \tag{7}$$

It is known that any linear dispersion code in the form of (6) can be expressed as $\mathbf{x}^- = \mathbf{G} \mathbf{s}^-$, where \mathbf{s}^- collects the real and imaginary parts of the symbols from the symbol vector $\mathbf{s} = [s_1, s_2, s_3, s_4]^T$.

The generator matrix G of the Golden code (\mathbf{X}_g) and the STBC presented in [8] has the property

$$\mathbf{G} \mathbf{G}^T = \mathbf{G}^T \mathbf{G} = \mathbf{I}_8 \tag{8}$$

Where \mathbf{I}_N denotes the $N \times N$ identity matrix. Therefore, the properties of the input signal s are not changed and the resulting STBC is said to have cubic shaping [3]. This also implies that the average power of the input symbol vector s remains unchanged whatever the properties of the signal. On the other hand, the property (11) is not satisfied with

X_{new}

$$\mathbf{G} \mathbf{G}^T = \mathbf{G}^T \mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 0 & \phi & -\psi & 0 & 0 \\ 0 & 1 & 0 & 0 & \psi & \phi & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \phi & -\psi \\ 0 & 0 & 0 & 1 & 0 & 0 & \psi & \phi \\ \phi & \psi & 0 & 0 & 1 & 0 & 0 & 0 \\ -\psi & \phi & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \phi & \psi & 0 & 0 & 1 & 0 \\ 0 & 0 & -\psi & \phi & 0 & 0 & 0 & 1 \end{bmatrix} \neq \mathbf{I}_8, \tag{9}$$

Where

$$\phi = \Re\{ab^*\} + \Re\{cd^*\} \text{ and } \psi = -\Re\{ab^*\} + \Re\{cd^*\}.$$

B. Reduced-Complexity ML-Detection

Before describing the reduced-complexity ML detection, we provide the following proposition. The idea of constructing such an STBC is based upon the following proposition

→ Any 2×2 matrix in the form of (5) with either $|a|=|c|$ or $|b|=|d|$ is ML detectable with exhaustive search complexity $O(M^2)$.

We provide the proof of this proposition considering the exhaustive search and then explain the corresponding reduced complexity SD.

1) Exhaustive Search: For the sake of simplicity, we first explain the interesting features of the code X_{new} given in (5) considering the exhaustive ML procedure. The exhaustive ML detector makes a search over all possible values of the transmitted symbols and decides in favor of (s_1, s_2, s_3, s_4) which minimizes the Euclidean distance $D(s_1, s_2, s_3, s_4)$ written as

$$D(s_1, s_2, s_3, s_4) = \sum_{k=1}^2 |y_{k1} - h_{k1}(as_1 + bs_3) - h_{k2}(as_2 + bs_4)|^2 + \sum_{l=1}^2 |y_{l2} + h_{l1}(cs_2^* + ds_4^*) - h_{l2}(cs_1^* + ds_3^*)|^2. \tag{10}$$

As explained above, an exhaustive search clearly involves the computation of M^4 metrics and M^4-1 comparisons, which is excessive for the 16-QAM and 64-QAM signal constellations. But the proposed STBC lends itself to a low-complexity implementation of the ML detector. In order to see this complexity reduction more clearly, we expand $D(s_1, s_2, s_3, s_4)$ and it is straightforward to show that the ML metric and it can be written as

$$\begin{aligned}
 D(s_1, s_2, s_3, s_4) &= C + g_1(s_1, s_3) + g_1(s_2, s_4) \\
 &+ \sum_{k=1}^2 2R \{h_{k1}h_{k2}^* (|a|^2 s_1s_2^* + ab^*s_1s_4^* + a^*bs_2^*s_3 + |b|^2 s_3s_4^*)\} \\
 &- \sum_{l=1}^2 2R \{h_{l1}h_{l2}^* (|c|^2 s_1s_2^* + c^*ds_1s_4^* + cd^*s_2^*s_3 + |d|^2 s_3s_4^*)\}
 \end{aligned}
 \tag{11}$$

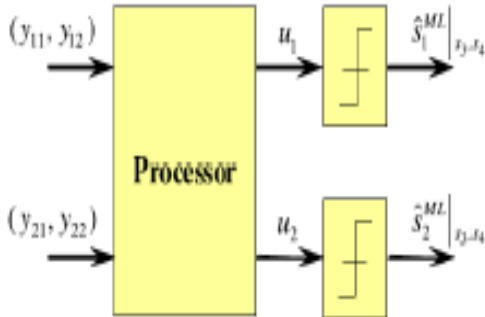


Fig:1 Processing of the received signals to determine the ML estimate of symbols s1 and s2 conditional on a particular combination of symbols s3 and s4.

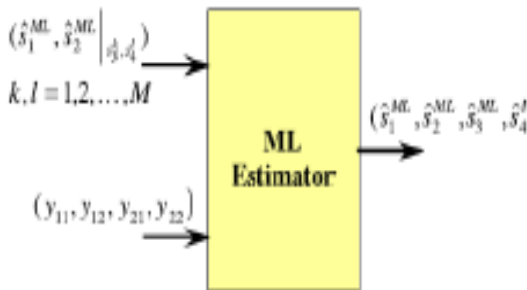


Fig: 2 Second Stage of Estimator

C. Parameter Optimization

Although the direct optimization of the design parameters a, b, c, d in the code matrix is infeasible especially for higher constellation sizes, the average transmit power constraints given in (6) allow a decrease in the number of parameters to be optimized. These equalities together with the constraint $|a| = |c|$ for optimal detectability lead immediately to the fact that all the design parameters should have the same magnitude, i.e., $|a| = |b| = |c| = |d| = 1/\sqrt{2}$. Now, without any loss of generality, we may set $a = c = 1/\sqrt{2}$. This decreases the number of unknown parameters without affecting the coding gain. Then, the remaining parameter pair (b, c) can be optimized numerically leading to a full-diversity scheme with large coding gain.

Note that the values of a and c affect the shape of the resulting lattice structure. Hence, depending on the constellation size, they can be optimized such that the number of nearest points (the so-called kissing number) is minimized. In order to set the values of the remaining parameters b and d, one may perform an exhaustive search so as to maximize the coding gain (and, thus, to ensure the full diversity) for QPSK signaling. This optimization leads to a set of parameter values which result in a coding gain of 2.

IV. RATE-3/4 2 x 2 STBC

The STBC given in (5) can be modified for a further reduction in the optimum detector complexity. More specifically, by setting $s_4 = s_3$ in (5) and scaling the energy of this symbol, we obtain the following 2×2 code with rate 3/4:

$$X_{new}^{3/4} = \begin{bmatrix} as_1 + bs_3/\sqrt{2} & -cs_2^* - ds_4^*/\sqrt{2} \\ as_2 + bs_4/\sqrt{2} & cs_1^* + ds_3^*/\sqrt{2} \end{bmatrix}$$

Where the notation $X_{3/4}$ new is used to distinguish the proposed code (5) from its reduced-rate version. In order to detect the symbols transmitted using , the full ML detector makes an exhaustive search over all possible values of the transmitted symbols and decides in favor of the triplet (s1, s2, s3) which minimizes the Euclidean distance that we denote by $D(s_1, s_2, s_3)$. Specifically, this exhaustive search involves the computation of M^3 metrics and $M^3 - 1$ comparison, which is also excessive for the 16-QAM and 64-QAM signal constellations. Now, dropping the symbol s4 lends itself to a lower-complexity implementation of the ML detector at the price of transmission rate reduction.

It can be seen that the signals $u_k, k = 1, 2$, will have only terms involving the respective symbol s_k and the estimation of symbols $s_k, k = 1, 2$, will benefit from full fourth-order spatial diversity. By sending the signals u_1 and u_2 to a threshold detector, we get the ML estimate of symbol s_1 and s_2 conditional only on the symbol s_3 . Optimization of the parameters in the reduced-rate case can be performed similarly to the full-rate case. The parameters a and c can be set to $1/\sqrt{2}$ without any loss of generality. In terms of the average transmitted power, the desired conditions can be expressed as and Using these constraints, we can easily obtain $|b| = |d| = 1/\sqrt{2}$. Then, an exhaustive search maximizing (3) gives a set of parameter pairs one of them being $b = d = (1+i\sqrt{7})/4$. It is also interesting to note that, by using the optimized values of the full-rate case, one can obtain the optimum values of the rate-3/4 case without any need for exhaustive search.

V. RESULTS

In this section, we present some comparisons between the aforementioned new STBCs and the existing alternatives. The simulations were carried out for the QPSK,

16-QAM and 64-QAM signal constellations, and the results are obtained for an Un-correlated Rayleigh fading channel with $E[|h_k|^2] = 1$ for all k, l . Two receive antennas were used in all cases.

A. Performance Comparison in the Full-Rate Case

We first give performance comparisons between the best known full-rate 2×2 STBCs and the proposed full-rate code (5). Fig. 3 shows the BER performance as a function of E_b/N_0 , where E_b denotes the average signal energy per bit, and provides comparisons between X_{new} , namely, the new STBC, and X_g (the Golden code). It can be seen that X_{new} achieves the same diversity gain and gives essentially the same results as X_g at substantially lower complexity.

The performance curves for the STBC proposed in [8] were not included in this figure, but we observed that they are quite indistinguishable from those of. Such comparisons also exist in [11] and coincide with our observations. Indeed, their conclusion is that the performance of is marginally inferior to that of [8] and very close to that of X_g .

The complexity reduction can be observed from Fig. 3, where the numbers of visited nodes are plotted as a function of SNR. Results in a considerable reduction in the number of computations. Since the number of visited nodes has a large impact on the required chip areas per throughput, these results indicate that enables to reduce the hardware complexity without any significant performance degradation. We now provide a performance comparison between namely, the proposed rate-3/4 STBC, and the two MIMO schemes in current mobile WiMAX system specifications (Alamouti's STBC and SM). With the optimized values, the proposed STBC maximizes the diversity gain and, therefore, it achieves the same BER curve slope as Alamouti's STBC with a constant coding gain independent of the constellation size. This is a crucial property as in the full-rate case, since we do not want vanishing determinants.

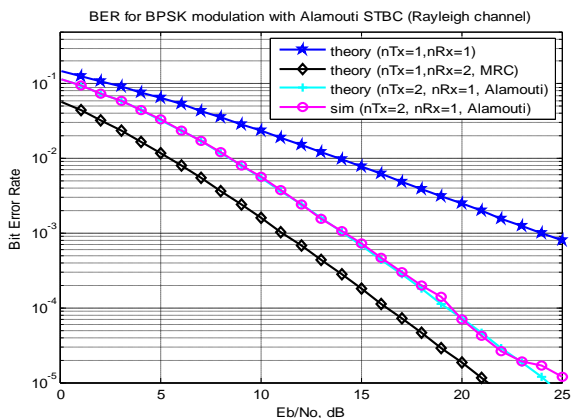


Fig:3 Comparison of 2Tx and 1Rx System BER

Performance using BPSK Modulation Based on Alamouti STBC.

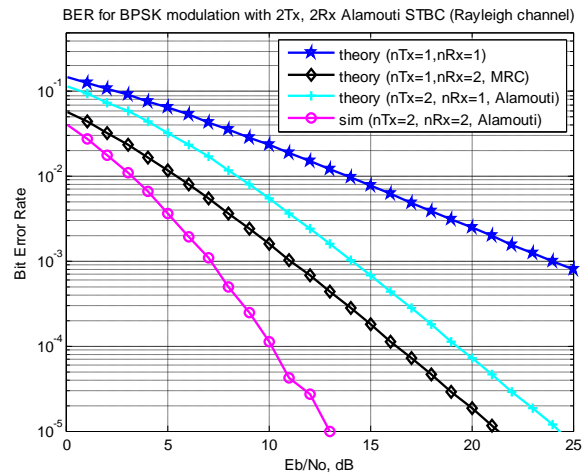


Fig:4 Comparison of 2Tx and 2Rx System BER Performance using BPSK Modulation Based on Alamouti STBC.

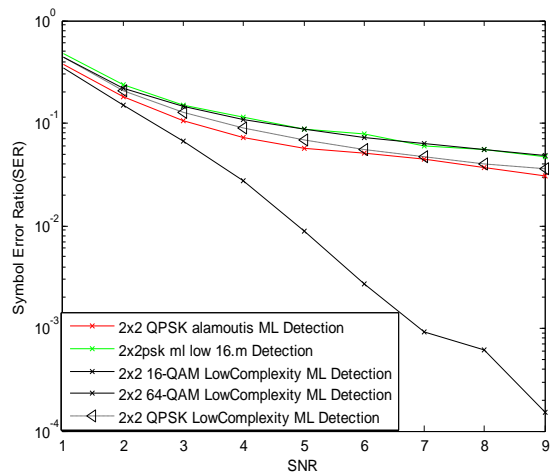


Fig:5 Comparison of 2Tx and 2Rx System SER Performance using ML Detection.

Alamouti scheme has the best BER performance, but also the lowest bit rate on a given channel bandwidth. The SM scheme doubles the bit rate, but it involves a strong SNR loss, which increases at lower BER values. As evidenced from these results, the proposed rate-3/4 scheme is an interesting alternative to these two MIMO schemes, as it substantially improves BER performance compared to SM, and it increases the bit rate by 25% compared to the Alamouti scheme at the price of some SNR loss.

A closer examination of the results shows that at the spectral efficiency of 3 bits per antenna use, the proposed technique outperforms Alamouti's STBC. Indeed, the new STBC with 16-QAM and Alamouti's STBC with 64-QAM have a spectral efficiency of 3 bits per antenna use, and the

results indicate that at the BER of 10^{-3} , the former outperforms the latter by 0.7 dB.

VI. CONCLUSION

In this paper, we have presented two new full-diversity 2×2 STBC designs with an inherent low-complexity optimum decoder. First, we have analyzed the proposed full-rate STBC and proved that it has full diversity with a non-vanishing coding gain. We have compared its performance and detection complexity to those of the Golden code, and the results indicated that the proposed scheme achieves the same performance while reducing the decoder complexity by orders of magnitude depending on the signal constellation

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